1. Consider the initial value problem

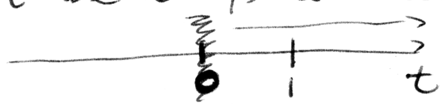
$$\begin{cases} t^2y'' + 3ty' + y = t \\ y(1) = -1, \quad y'(1) = 0 \end{cases}$$

a. (5 pts) Determine the longest interval in which the Initial Value Problem is certain to have a unique solution.

$t^2 = 0$ @ $t = 0$

$3t, 1, \& \ t$ are always continuous

$\boxed{t > 0}$



b. (10 pts) Show that $y_1(t) = t^{-1}$ and $y_2(t) = t^{-1}\ln(t)$ form **a fundamental set of solutions** of

$$t^2y'' + 3ty' + y = 0$$

I

$$\begin{aligned} 1\,&(y_1 = t^{-1}) \\ 3t\,&(y_1' = -t^{-2}) \\ +\ t^2\,&(y_1'' = 2t^{-3}) \\ \hline 0 &= 0 \ \checkmark \end{aligned} \qquad \begin{aligned} 1\,&(y_2 = t^{-1}\log t) \\ 3t\,&(y_2' = -t^{-2}\log t + t^{-2}) \\ +\ t^2\,&(y_2'' = 2t^{-3}\log t - 3t^{-3}) \\ \hline 0 &= 0 \ \checkmark \end{aligned}$$

II

$$w(t^{-1}, t^{-1}\log t) = \begin{vmatrix} t^{-1} & t^{-1}\log t \\ -t^{-2} & -t^{-2}\log t + t^{-2} \end{vmatrix} = t^{-3}$$

not always 0

I $\Rightarrow$ y_1 & y_2 are sol'ns

II $\Rightarrow$ y_1 & y_2 are indep.

I & II $\Rightarrow$ y_1 & y_2 are a fund. sol'n set

Yes, Virginia, this is INHOMOGENEOUS

c. (15) Using **1.b**, find the **general solution** of $t^2y'' + 3ty' + y \neq \boxed{t.}$

Then, solve the **initial value problem** $\begin{cases} t^2y'' + 3ty' + y = t \\ y(1) = -1, \quad y'(1) = 0 \end{cases}$

It also has non-constant coefficients!

$$y_H = C_1 y_1 + C_2 y_2 \qquad\qquad y = y_H + y_I$$

$$y_I = u_1 y_1 + u_2 y_2$$

$$u_1 = -\int \frac{y_2 \cdot \text{RHS}}{W(y_1, y_2)}\, dt$$

in STD form! RHS $= t/t^2$

$$= -\int \frac{t^{-1}\log t \cdot t^{-1}}{t^{-3}}\, dt$$

$$= -\int t \log t = -\left[\frac{1}{2}t^2 \log t - \frac{1}{4}t^2\right]$$

$$u_2 = \int \frac{y_1 \cdot \text{RHS}}{W(y_1, y_2)}\, dt = \int \frac{t^{-1} \cdot t^{-1}}{t^{-3}}\, dt = \int t\, dt = \frac{1}{2}t^2$$

$$y_I = -\left(\frac{1}{2}t^2 \log t - \frac{1}{4}t^2\right)\left(t^{-1}\right) + \left(\frac{1}{2}t^2\right)\left(t^{-1}\log t\right)$$

$$y_I = \frac{1}{4}t$$

$$y = C_1 t^{-1} + C_2 t^{-1}\log t + \frac{1}{4}t$$ GEN'L SOL'N

$$y' = -C_1 t^{-2} - C_2 t^{-2}\log t + C_2 t^{-2} + \frac{1}{4}$$

$$\left.\begin{aligned} -1 = y(1) &= C_1 + 1/4 \\ 0 = y(0) &= -C_1 + C_2 + 1/4 \end{aligned}\right\} \Rightarrow \begin{cases} C_1 = -5/4 \\ C_2 = -3/2 \end{cases}$$

$$y = -\frac{5}{4}t^{-1} - \frac{3}{2}t^{-1}\log t + \frac{1}{4}t$$ IVP SOL'N

2. a. (20 pts) Solve the following equation by means of a power series. Find the recurrence relation, and give the **first four terms in each of two linearly independent solutions**.

$$(1 + x^2)y'' + 2xy' - 2y = 0$$

$$y = \sum_0^\infty a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6 + \dots$$

$$y' = \sum_1^\infty n a_n x^{n-1}$$

$$y'' = \sum_2^\infty n(n-1) a_n x^{n-2}$$

$$\sum_2^\infty n(n-1)a_n x^{n-2} + \sum_2^\infty n(n-1)a_n x^n + \sum_1^\infty 2na_n x^n + \sum_0^\infty -2a_n x^n = 0$$

$$\sum_0^\infty \left[(n+2)(n+1)a_{n+2} + n(n-1)a_n + 2na_n - 2a_n \right] x^n = 0$$

$$\Rightarrow (n+2)(n+1)a_{n+2} + \left[n(n-1) + 2n - 2\right] a_n = 0 \quad \forall n \geq 0$$

$$a_{n+2} = \frac{-n^2 - n + 2}{(n+2)(n+1)} \cdot a_n \quad \forall n \geq 0$$ RECURRENCE REL.

$$a_{n+2} = \left(\frac{1-n}{1+n}\right) a_n \quad \forall n \geq 0 \; : \quad a_0,\; a_1,\; a_2 = a_0,\; a_3 = 0 \cdot a_1 = 0$$

$$a_4 = -\frac{1}{3}a_2 = -\frac{1}{3}a_0,\; a_5 = 0 \cdot a_3 = 0,$$

In fact, $a_n = 0$ for all odd $n \geq 3$ $\longrightarrow a_5 = 0 \cdot a_3 = 0,\; a_6 = -\frac{3}{5}a_4 = \frac{1}{5}a_0$

$$y = a_0 + a_1 x + a_0 x^2 + 0x^3 - \frac{1}{3}a_0 x^4 + 0x^5 + \frac{1}{5}a_0 x^6 + \dots$$

$$y = a_0\left(1 + x^2 - \frac{1}{3}x^4 + \frac{1}{5}x^5 + \dots\right) + a_1(x)$$

$$\Rightarrow \quad y_1 = 1 + x^2 - \frac{1}{3}x^4 + \frac{1}{5}x^5 + \dots$$

$$y_2 = x$$

TWO INDEP. SOL'NS

b. (10 pts) Use **2.a.** to solve the following initial value problem:

$$(1+x^2)y'' + 2xy' - 2y = 0, \qquad y(0)=0, \quad y'(0)=1.$$

HOMOG. & LINEAR $\Rightarrow$ GEN'L SOL'N is

$$y = a_0 y_1 + a_1 y_2$$

$$y' = a_0 y_1' + a_1 y_2'$$

$$0 = a_0 \underbrace{y_1(0)}_{1} + a_1 \underbrace{y_2(0)}_{0}$$

$$1 = a_0 \underbrace{y_1'(0)}_{0} + a_1 \underbrace{y_2'(0)}_{1}$$

$\Rightarrow a_0 = 0$ & $a_1 = 1$

$\Rightarrow$ $\boxed{y = x \quad \text{IVP SOL'N}}$

FIRST ORDER; NOT SEPARABLE; NOT LINEAR

3. (15 pts) Find the solution of the following **initial value problem:**

$$\begin{cases} -t^3 + \dfrac{y}{t} + (y + \ln t)y' = 0, \\ y(1) = 1. \end{cases}$$

$$\left(-t^3 + y/t\right) dt + \left(y + \log t\right) dy = 0$$

$$dF = \frac{\partial F}{\partial t}\, dt + \frac{\partial F}{\partial y}\, dy \quad \text{if } F = F(t,y)$$

$$\frac{\partial^2 F}{\partial t \partial y} = 1/t = \frac{\partial^2 F}{\partial y \partial t} \Rightarrow \text{DE is exact}$$

$$\frac{\partial F}{\partial t} = -t^3 + y/t$$

$$F = -\frac{1}{4}t^4 + y \log t + C(y)$$

$$\frac{\partial F}{\partial y} = \log t + C'(y) = y + \log t$$

$$C'(y) = y$$

OR

$$\frac{\partial F}{\partial y} = y + \log t$$

$$F = \frac{1}{2}y^2 + y \log t + D(t)$$

$$\frac{\partial F}{\partial t} = y/t + D'(t) = -t^3 + y/t$$

$$D'(t) = -t^3$$

$$\Rightarrow \quad -t^4/4 + y \log t + y^2/2 = E \quad \text{is a sol'n to the DE}$$

IC: $y(1) = 1$

$$-1/4 + 1 \cdot \log 1 + 1/2 = E \Rightarrow E = 1/4$$

$$\Rightarrow -t^4/4 + y \log t + y^2/2 = 1/4$$

NB:

$$y = -\log t + \sqrt{(\log t)^2 + t^4/2 + 1/2}$$

the "−" root doesn't match the IC

4. (20 pts) Find the **general solution** of the system of equations:

$$\vec{x}'(t) = \begin{pmatrix} 3 & 4 \\ 3 & 2 \end{pmatrix} \vec{x}(t)$$

Then, solve the Initial Value Problem $\vec{x}'(t) = \begin{pmatrix} 3 & 4 \\ 3 & 2 \end{pmatrix} \vec{x}(t), \quad \vec{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\det \begin{pmatrix} 3-r & 4 \\ 3 & 2-r \end{pmatrix} = 0 \Rightarrow r^2 - 5r - 6 = 0$$

$$(r-6)(r+1) = 0$$

$$r^{(1)} = 6 \ \& \ r^{(2)} = -1$$

$$\begin{pmatrix} -3 & 4 \\ 3 & -4 \end{pmatrix} \xi^{(1)} = \bar{0} \Rightarrow -3\xi_1^{(1)} + 4\xi_2^{(1)} = 0 \Rightarrow \xi^{(1)} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} c_1$$

$$\begin{pmatrix} 4 & 4 \\ 3 & 3 \end{pmatrix} \xi^{(2)} = \bar{0} \Rightarrow \xi_1^{(2)} + \xi_2^{(2)} = 0 \Rightarrow \xi^{(2)} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} c_2$$

$$\bar{x} = c_1 \begin{pmatrix} 4 \\ 3 \end{pmatrix} e^{6t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t}$$ GEN'L SOL'N

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \bar{x}(0) = c_1 \begin{pmatrix} 4 \\ 3 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\left.\begin{aligned} 1 &= 4c_1 + c_2 \\ 1 &= 3c_1 - c_2 \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} c_1 &= 2/7 \\ c_2 &= -1/7 \end{aligned}\right.$$

$$\bar{x} = (2/7) \begin{pmatrix} 4 \\ 3 \end{pmatrix} e^{6t} + (-1/7) \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t}$$ IVP SOL'N

$$\bar{x} = \begin{pmatrix} 8/7\, e^{6t} - 1/7\, e^{-t} \\ 6/7\, e^{6t} + 1/7\, e^{-t} \end{pmatrix}$$

5. (15 pts) Find the general solution of the system of equations:

$$\vec{x}'(t) = \begin{pmatrix} 3 & -1 \\ 4 & 3 \end{pmatrix} \vec{x}(t)$$

$$\det \begin{pmatrix} 3-r & -1 \\ 4 & 3-r \end{pmatrix} = 0$$

$$r^2 - 6r + 13 = 0$$

$$(r-3)^2 + 2^2 = 0$$

$$r = 3 \pm 2i$$

$r = 3+2i$

$$\begin{pmatrix} -2i & -1 \\ 4 & -2i \end{pmatrix} \bar{\xi} = \bar{0} \Rightarrow \xi_2 = -2i\xi_1 \Rightarrow \bar{\xi} = \begin{pmatrix} 1 \\ -2i \end{pmatrix} c_1$$

$$\bar{x} = c_1 \begin{pmatrix} 1 \\ -2i \end{pmatrix} \exp((3+2i)t) + c_2 \begin{pmatrix} 1 \\ 2i \end{pmatrix} \exp((3-2i)t)$$

$$\begin{pmatrix} 1 \\ -2i \end{pmatrix} (\cos 2t + i \sin 2t) = \begin{pmatrix} \cos 2t & + i \sin 2t \\ 2\sin 2t & -2i\cos 2t \end{pmatrix}$$

$$\bar{x} = D_1 e^{3t} \begin{pmatrix} \cos 2t \\ 2\sin 2t \end{pmatrix} + D_2 e^{3t} \begin{pmatrix} \sin 2t \\ -2\cos 2t \end{pmatrix}$$

6. (20 pts) Solve the following initial value problem

$$\begin{cases} y'' + 4y = e^{-t} - u_{\pi/2}(t)e^{-(t-\pi/2)} \\ y(0) = -1, \quad y'(0) = 0 \end{cases}$$

If $Y = \mathcal{L}(y)$,

$$\left(s^2Y - s(-1) - (0)\right) + 4Y = \frac{1}{s+1} - e^{-\frac{\pi}{2}s} \cdot \frac{1}{s+1}$$

$$Y = -\frac{s}{s^2+4} + \frac{1}{(s^2+4)(s+1)} - e^{-\frac{\pi}{2}s} \cdot \frac{1}{(s^2+4)(s+1)}$$

$$\frac{1}{(s^2+4)(s+1)} = \frac{As+B}{s^2+4} + \frac{C}{s+1}$$

$C = 1/5 \quad (s=-1)$

$1/4 = B/4 + C \quad (s=0)$

$B = 1/5$

$\frac{1}{10} = \frac{A+B}{5} + \frac{C}{2} \quad (s=1)$

$A = -1/5$

$$Y = \underbrace{-\frac{s}{s^2+4} - \frac{1}{5}\frac{s}{s^2+4}}_{-\frac{6}{5}\frac{s}{s^2+4}} + \underbrace{\frac{1}{5}\frac{1}{s^2+4}}_{\frac{1}{10}\frac{2}{s^2+4}} + \frac{1}{5}\frac{1}{s+1} - e^{-\frac{\pi}{2}s}\left(-\frac{1}{5}\frac{s}{s^2+4} + \frac{1}{5}\frac{1}{s^2+4} + \frac{1}{5}\frac{1}{s+1}\right)$$

$$y = -\frac{6}{5}\cos(2t) + \frac{1}{10}\sin(2t) + \frac{1}{5}e^{-t} - u_{\pi/2}(t)\left[-\frac{1}{5}\cos\left(2\left(t-\frac{\pi}{2}\right)\right) + \frac{1}{10}\sin\left(2\left(t-\frac{\pi}{2}\right)\right) + \frac{1}{5}e^{-(t-\pi/2)}\right]$$

$$y = -\frac{6}{5}\cos(2t) + \frac{1}{10}\sin(2t) + \frac{1}{5}e^{-t} + u_{\pi/2}(t)\left[\frac{1}{5}\cos(2t) - \frac{1}{10}\sin(2t) + \frac{e^{\pi/2}}{5}e^{-t}\right]$$

FIRST ORDER, NOT SEPARABLE, NOT EXACT, LINEAR W/CONST COEFFS & SPEC. RHS

7. (15 pts) Solve the following Initial Value Problem:

$$\begin{cases} y' - 2y = \cos(t) \\ y(0) = -2 \end{cases}$$

(I) UNDET COEFFS

y_H: $\quad r - 2 = 0$

$$y_H = Ce^{2t}$$

$$-2\left(y_I = A\cos t + B\sin t\right)$$

$$+1\left(y_I' = B\cos t - A\sin t\right)$$

$$\cos t = (B-2A)\cos t + (-A-2B)\sin t$$

$$\left.\begin{aligned} -2A + B &= 1 \\ -A - 2B &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} A &= -2/5 \\ B &= 1/5 \end{aligned}$$

$$y = y_H + y_I = Ce^{2t} - \frac{2}{5}\cos t + \frac{1}{5}\sin t$$

$$-2 = y(0) = C - 2/5 \quad \Rightarrow \quad C = -8/5$$

$$y = -\frac{8}{5}e^{2t} - \frac{2}{5}\cos t + \frac{1}{5}\sin t$$

(II) LAPLACE TRANSFORMS $\quad$ w/ $\mathcal{L}(y) = Y$

$$sY + 2 - 2Y = \frac{s}{s^2+1}$$

$$Y = -\frac{2}{s-2} + \frac{s}{(s^2+1)(s-2)}$$

$$\frac{s}{(s^2+1)(s-2)} = \frac{As+B}{s^2+1} + \frac{C}{s-2} \Rightarrow C = 2/5 \quad B = 1/5 \quad A = -2/5$$

(try $s = 2, 0, 1$ or $\underset{s^2}{A + C = 0}$, $\underset{s}{-2A + B = 1}$, $\underset{1}{-2B + C = 0}$)

$$Y = -\frac{8}{5}\frac{1}{s-2} - \frac{2}{5}\frac{s}{s^2+1} + \frac{1}{5}\frac{1}{s^2+1}$$

$$y = -\frac{8}{5}e^{2t} - \frac{2}{5}\cos t + \frac{1}{5}\sin t$$

(III) INTEG. FACTOR / VAR OF PARAMS

$$y' - 2y = \cos t$$

$$\text{IF} = \exp\left(\int -2\,dt\right) = e^{-2t}$$

$$\frac{d}{dt}\left(e^{-2t}y\right) = e^{-2t}\cos t$$

$$\int e^{-2t}\cos t\,dt = e^{-2t}\sin t + 2\left[-e^{-2t}\cos t - 2\int e^{-2t}\cos t\,dt\right]$$

IBP (either/or)

$$= -\frac{1}{2}e^{-2t}\cos t - \frac{1}{2}\left[-\frac{1}{2}e^{-2t}\sin t + \frac{1}{2}\int e^{-2t}\cos t\,dt\right]$$

$$= \frac{1}{5}e^{-2t}\sin t - \frac{2}{5}e^{-2t}\cos t + C$$

$$\Rightarrow e^{-2t}y = \frac{1}{5}e^{-2t}\sin t - \frac{2}{5}e^{-2t}\cos t + C$$

$$y = \frac{1}{5}\sin t - \frac{2}{5}\cos t + Ce^{2t}$$

$$-2 = y(0) = -\frac{2}{5} + C \Rightarrow C = -8/5$$

$$y = \frac{1}{5}\sin t - \frac{2}{5}\cos t - \frac{8}{5}e^{2t}$$

FIRST ORDER, LINEAR & EXACT

8. (15 pts) Find the **general solution** of the following differential equation:

$$2\cos(2t)\,y + \sin(2t)\,y' = 2t^2$$

$$\frac{d}{dt}\left(y\sin(2t)\right) = 2t^2$$

$$y\sin(2t) = \frac{2}{3}t^3 + C$$

$$y = \frac{2}{3}t^3\csc(2t) + C\csc(2t)$$

STANDARD FORM:

$$y' + 2\cot(2t)\,y = 2t^2\sec(2t)$$

$$IF = \exp\left(\int 2\cot(2t)\,dt\right)$$

$$= \exp\left(\log|\sin(2t)|\right)$$

$$= \sin(2t)$$

$$\sin(2t)\,y' + 2\cos(2t)\,y = 2t^2$$

9. (20 pts) Find the Fourier Series for the function defined by

$$f(x) = \begin{cases} 0 & -2 \le x < 0 \\ x & 0 \le x < 2 \end{cases}$$

and $f(x+4) = f(x)$. $\Rightarrow$ $L = 2$ (half-period)

$$f(x) = a_0/2 + \sum_1^\infty a_n \cos\left(\frac{n\pi x}{2}\right) + b_n \sin\left(\frac{n\pi x}{2}\right)$$

where

$$a_n = \frac{1}{2}\int_{-2}^{2} f(x) \cos\left(\frac{n\pi x}{2}\right) dx \qquad n \ge 0$$

$$a_n = \frac{1}{2}\int_{0}^{2} x \cos\left(\frac{n\pi x}{2}\right) dx \qquad n \ge 0 \qquad (f(x) = 0 \text{ for } -2 \le x \le 0)$$

$$a_n = \frac{1}{2}\left[(x)\left(\frac{2}{n\pi}\right)\sin\left(\frac{n\pi x}{2}\right) + \left(\frac{2}{n\pi}\right)^2 \cos\left(\frac{n\pi x}{2}\right)\right]_0^2 \qquad n \ge 1$$

$$a_n = \frac{1}{2}\left[\left(\frac{2}{n\pi}\right)^2 (-1)^n - \left(\frac{2}{n\pi}\right)^2\right] \qquad n \ge 1$$

$$a_0 = \frac{1}{2}\int_0^2 x\, dx = \frac{1}{2}\left[\frac{x^2}{2}\right]_0^2 = 1$$

$$b_n = \frac{1}{2}\int_0^2 x \sin\left(\frac{n\pi x}{2}\right) dx$$

$$= \frac{1}{2}\left[(-x)\left(\frac{2}{n\pi}\right)\cos\left(\frac{n\pi x}{2}\right) + \left(\frac{2}{n\pi}\right)^2 \sin\left(\frac{n\pi x}{2}\right)\right]_0^2$$

$$= \frac{1}{2}\left[-\frac{4}{n\pi}(-1)^n\right]$$

$$\boxed{f(x) = \frac{1}{2} + \sum_1^\infty \frac{2}{n^2\pi^2}\left((-1)^n - 1\right)\cos\left(\frac{n\pi x}{2}\right) - \frac{2}{n\pi}(-1)^n \sin\left(\frac{n\pi x}{2}\right)}$$

10. (20 pts) Solve the following Initial Value Problem:

$$\begin{cases} y^{(3)} - y'' = t + e^{-t} \\ y(0) = 1, \quad y'(0) = 0, \quad y''(0) = 1 \end{cases}$$

y_H: $\quad r^3 - r^2 = 0$

$$r = 0, 0, 1$$

$$y_H = C + Dt + Ee^t$$

y_I: $\quad y_I = (At + B)t^2$

$$y_I' = 3At^2 + 2Bt$$

$$y_I'' = 6At + 2B$$

$$y_I''' = 6A$$

$$t = y''' - y''$$

$$t = 6A - (6At + 2B)$$

$$\Rightarrow \left.\begin{aligned} 0 &= 6A - 2B \\ 1 &= -6A \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} A &= -1/6 \\ B &= -1/2 \end{aligned}\right.$$

$$y_I = -\frac{1}{6}t^3 - \frac{1}{2}t^2$$

y_{II}: $\quad y_{II} = Ae^{-t}$

$$y_{II}' = -Ae^{-t}$$

$$y_{II}'' = Ae^{-t}$$

$$y_{II}''' = -Ae^{-t}$$

$$e^{-t} = (-Ae^{-t}) - (Ae^{-t}) \Rightarrow 1 = -2A \Rightarrow A = -1/2$$

$$y_{II} = -\frac{1}{2}e^{-t}$$

$$y = y_H + y_I + y_{II}$$

$$y = C + Dt + Ee^t - \frac{1}{6}t^3 - \frac{1}{2}t^2 - \frac{1}{2}e^{-t}$$

$$y' = D + Ee^t - \frac{1}{2}t^2 - t + \frac{1}{2}e^{-t}$$

$$y'' = Ee^t - t - 1 - \frac{1}{2}e^{-t}$$

$$\left.\begin{aligned} 1 &= y(0) = C + E - 1/2 \\ 0 &= y'(0) = D + E + 1/2 \\ 1 &= y''(0) = E - 1 - 1/2 \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} C &= -1 \\ D &= -3 \\ E &= \frac{5}{2} \end{aligned}\right.$$

$$y = -1 - 3t - \frac{1}{2}t^2 - \frac{1}{6}t^3 + \frac{5}{2}e^t - \frac{1}{2}e^{-t}$$

ALT: $\mathcal{L}(y) = Y$

$$(s^3Y - s^2 - 1) - (s^2Y - s) = \frac{1}{s^2} + \frac{1}{s+1}$$

$$Y = \frac{s^5 + 2s^2 + s + 1}{s^4(s-1)(s+1)}$$

$$Y = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s^4} + \frac{E}{s-1} + \frac{F}{s+1}$$

$$F = -\frac{1}{2} \quad E = \frac{5}{2} \quad D = -1 \quad \ldots$$