§1.3

(2) 2nd order

nonlinear ($(1+y^2)y''$ term)

(3) 4th order

linear

(6) 3rd order

linear

(8) $y'' + 2y' - 3y = 0$

$y_1 = e^{-3t}$ $\qquad$ $y_2 = e^{t}$

$y_1' = -3e^{-3t}$ $\qquad$ $y_2' = e^{t}$

$y_1'' = 9e^{-3t}$ $\qquad$ $y_2'' = e^{t}$

so that

$9e^{-3t} - 6e^{-3t} - 3e^{-3t} = 0$ ✓ $\qquad$ $e^{t} + 2e^{t} - 3e^{t} = 0$ ✓

(10) $y^{(4)} + 4y^{(3)} + 3y = t$

$y_1 = t/3$

$y_1' = 1/3$

$y_1'' = y_1^{(3)} = y_1^{(4)} = 0$

$y_2 = e^{-t} + t/3$

$y_2' = -e^{-t} + 1/3$

$y_2'' = e^{-t}$

$y_2^{(3)} = -e^{-t}$

$y_2^{(4)} = e^{-t}$

so that

$0 + 4 \cdot 0 + 3(t/3) = t$ $\checkmark$

$e^{-t} - 4e^{-t} + 3(e^{-t} + t/3) = t$ $\checkmark$

(12) $t^2 y'' + 5ty' + 4y = 0$

$y_1 = t^{-2}$

$y_1' = -2t^{-3}$

$y_1'' = 6t^{-4}$

$y_2 = t^{-2} \log t$

$y_2' = -2t^{-3} \log t + t^{-3}$

$y_2'' = 6t^{-4} \log t - 5t^{-4}$

$6t^{-2} - 10t^{-2} + 4t^{-2} = 0$ $\checkmark$

$6t^{-2} \log t - 5t^{-2} - 10t^{-2} \log t + 5t^{-2} + 4t^{-2} \log t = 0$ $\checkmark$

(17) $y'' + y' - 6y = 0$

$y = e^{rt}$

$y' = re^{rt}$

$y'' = r^2 e^{rt}$

$r^2 e^{rt} + re^{rt} - 6e^{rt} = 0$

$\Rightarrow (r^2 + r - 6)e^{rt} = 0$

$\Rightarrow r^2 + r - 6 = 0$ <u>or</u> $e^{rt} = 0$

$\Downarrow$ $\Downarrow$

$\boxed{r = -3 \text{ or } r = 2}$ never (no r & t)

§2.1

(1) $y' + 3y = t + e^{-2t}$ $IF = \exp\left(\int 3\,dt\right) = e^{3t}$

$\frac{d}{dt}(e^{3t}y) = te^{3t} + e^{t}$

$e^{3t}y = \frac{1}{3}te^{3t} - \frac{1}{9}e^{3t} + e^{t} + C$

$\Rightarrow y = \frac{1}{3}t - \frac{1}{9} + e^{-2t} + Ce^{-3t}$

① (cont)

$$\lim_{t\to\infty} y = +\infty$$ ← no matter C

or (if you're smart)

$$\lim_{t\to\infty}\left(y-\left(\tfrac{1}{3}t-\tfrac{1}{9}\right)\right)=0$$

so that as $t\to\infty$ then $y\to\infty$

or $y-\left(\frac{1}{3}t-\frac{1}{9}\right)\to 0$

(or $y\longrightarrow \frac{1}{3}t-\frac{1}{9}$)

③ $y'-2y=3e^{t}$

$y'+(-2)y=3e^{t}$ IF $=\exp\left(\int -2\,dt\right)=e^{-2t}$

$$\frac{d}{dt}\left(e^{-2t}y\right)=3e^{-t}$$

$$e^{-2t}y=-3e^{-t}+C$$

$$y=-3e^{t}+Ce^{2t}$$

$\lim\limits_{t\to\infty} y=\pm\infty$ depending on the sign of C

(e^{2t} grows faster than e^{t};

$-3e^{t}+Ce^{2t}=e^{2t}(\underbrace{-3e^{-t}}_{\to 0}+C)$)

(6) $ty' + 2y = \sin t \qquad t > 0$

$$y' + \frac{2}{t} y = \frac{\sin t}{t} \qquad IF = \exp\left(\int \frac{2}{t}\,dt\right)$$

$$\frac{d}{dt}\left(t^2 y\right) = t \sin t \qquad = \exp\left(2 \log t\right)$$

$$= t^2$$

$$t^2 y = -t \cos t + \sin t + C$$

$$y = -\frac{\cos t}{t} + \frac{\sin t}{t^2} + \frac{C}{t^2}$$

$$\lim_{t \to \infty} y = 0 \qquad \text{(no matter } c\text{)}$$

(13) $y' - y = 2te^{2t} \qquad y(0) = 1$

$$y' + (-1) y = 2te^{2t} \qquad IF = \exp\left(\int -1\,dt\right) = e^{-t}$$

$$\frac{d}{dt}\left(e^{-t} y\right) = 2te^{t}$$

$$e^{-t} y = 2te^{t} - 2e^{t} + C$$

$$y = 2te^{2t} - 2e^{2t} + Ce^{t}$$

$$1 = y(0) = 0 \quad 2 + C \implies C = +3$$

so $y = 2te^{2t} - 2e^{2t} + 3e^{t}$

(20) $ty' + (t+1)y = t \qquad y(\log 2) = 1$

$(t>0) \quad y' + (1 + 1/t)\, y = 1 \qquad IF = \exp\left(\int (1 + 1/t)\, dt\right)$

$$= \exp(t + \log t)$$

$$= te^t$$

$$\frac{d}{dt}\left(te^t y\right) = te^t$$

$$te^t y = te^t - e^t + C$$

$$y = 1 - \frac{1}{t} + \frac{C}{te^t}$$

$$1 = y(\log 2) = 1 - \frac{1}{\log 2} + \frac{C}{(\log 2)\exp(\log 2)}$$

$$1 = 1 - \frac{1}{\log 2} + \frac{C}{2\log 2}$$

$$1 = 1 + \frac{C-2}{2\log 2} \Rightarrow \frac{C-2}{2\log 2} = 0$$

$$\Rightarrow C = 2$$

so $y = 1 - \frac{1}{t} + \frac{2}{te^t}$

(28) IN CLASS