§3.1

③ $6y'' - y' - y = 0$

$6r^2 - r - 1 = 0$

$(2r-1)(3r+1) = 0$

$r = 1/2, -1/3$

$y = Ce^{t/2} + De^{-t/3}$

⑤ $y'' + 5y' = 0$

$r^2 + 5r = 0$

$r(r+5) = 0$

$r = 0, -5$

$y = C + De^{-5t}$ (NB: $e^{0t} = e^0 = 1$)

(10) $y'' + 4y' + 3y = 0$ $\qquad y(0) = 2$

$r^2 + 4r + 3 = 0$ $\qquad y'(0) = -1$

$(r+1)(r+3) = 0$

$r = -1, -3$

$y = Ce^{-t} + De^{-3t}$

$y' = -Ce^{-t} - 3De^{-3t}$

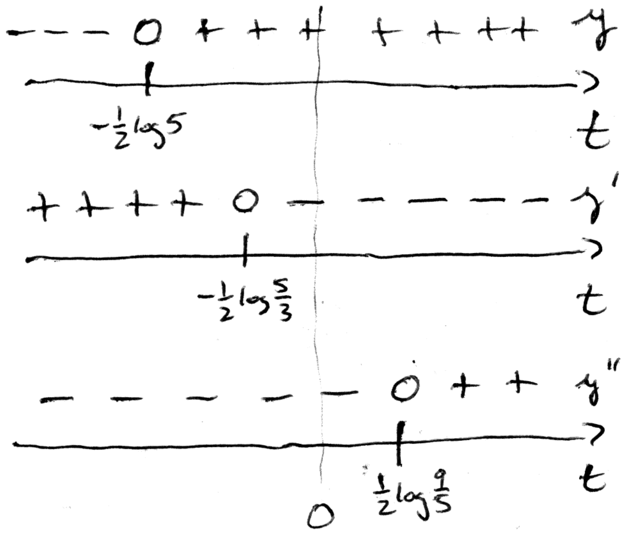
$2 = y(0) = C + D$

$-1 = y'(0) = -C - 3D$

$C = 5/2 \quad D = -1/2$

$y = 5/2\, e^{-t} - 1/2\, e^{-3t}$

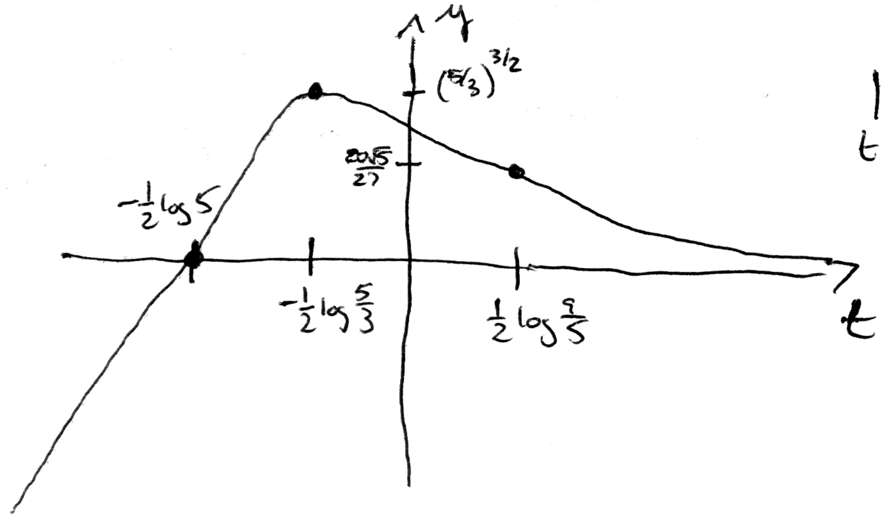
(10) (cont)




$$y = \frac{5}{2}e^{-t} - \frac{1}{2}e^{-3t}$$

$$y' = -\frac{5}{2}e^{-t} + \frac{3}{2}e^{-3t}$$

$$y'' = \frac{5}{2}e^{-t} - \frac{9}{2}e^{-3t}$$




$$\lim_{t\to\infty} y = 0$$

(11) $6y'' - 5y' + y = 0$ $\qquad y(0) = 4$

$\qquad\qquad\qquad\qquad\qquad\quad\; y'(0) = 0$

$$6r^2 - 5r + 1 = 0$$

$$(3r - 1)(2r - 1) = 0$$

$$r = 1/3,\ 1/2$$

$$y = Ce^{t/3} + De^{t/2}$$

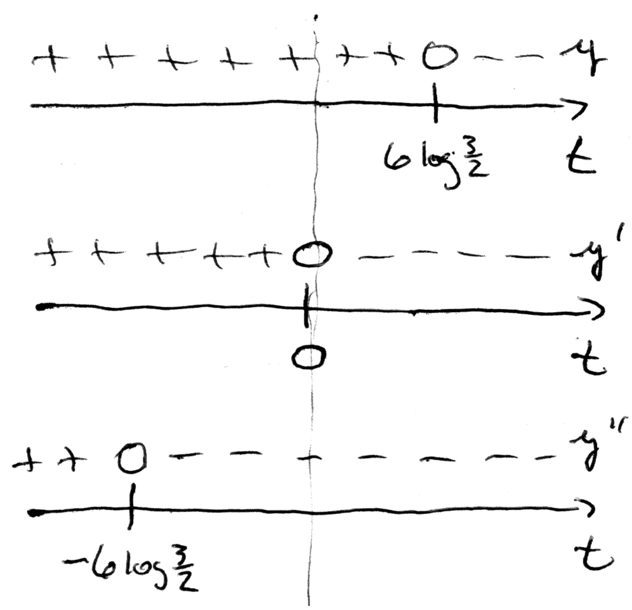
$$y' = C/3\, e^{t/3} + D/2\, e^{t/2}$$

$$4 = y(0) = C + D$$

$$0 = y'(0) = C/3 + D/2$$

$$C = 12 \qquad D = -8$$

$$y = 12e^{t/3} - 8e^{t/2}$$

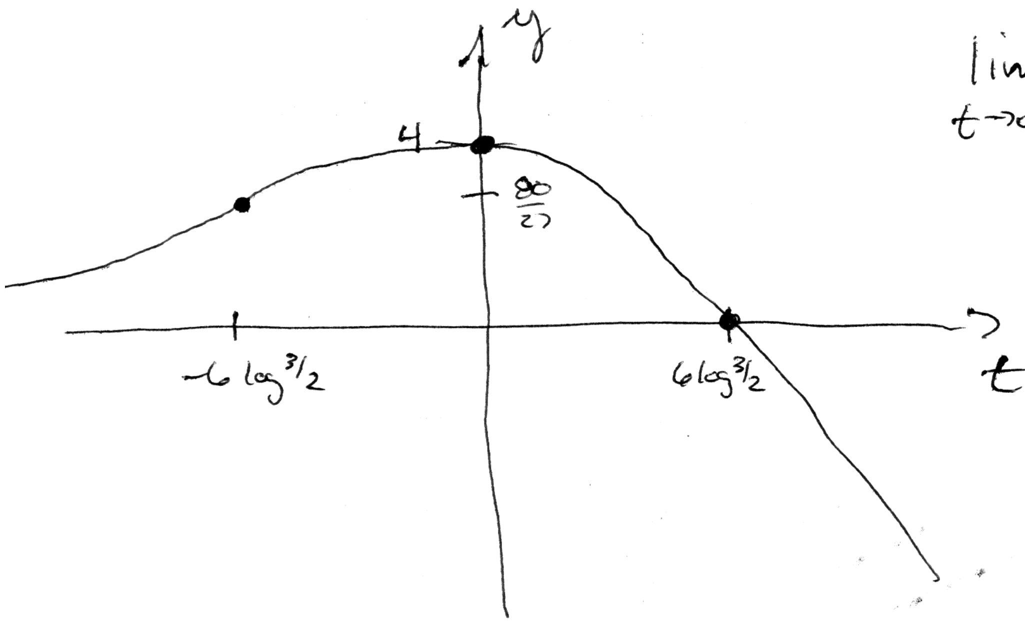



$$y = 12e^{t/3} - 8e^{t/2}$$

$$y' = 4e^{t/3} - 4e^{t/2}$$

$$y'' = \frac{4}{3}e^{t/3} - 2e^{t/2}$$

(11) (cont)




$$\lim_{t\to\infty} y = -\infty$$

(13)

$$y'' + 5y' + 3y = 0 \qquad y(0) = 1$$

$$r^2 + 5r + 3 = 0 \qquad y'(0) = 0$$

$$r = -\frac{5}{2} \pm \frac{\sqrt{13}}{2} = -\alpha, -\bar{\alpha} \quad \text{if} \quad \alpha = \frac{5+\sqrt{13}}{2}$$

$$\bar{\alpha} = \frac{5-\sqrt{13}}{2}$$

$$y = Ce^{-\alpha t} + De^{-\bar{\alpha} t}$$

$$y' = -\alpha C e^{-\alpha t} - \bar{\alpha} D e^{-\bar{\alpha} t}$$

NB: $\frac{1}{\alpha} = \frac{\bar{\alpha}}{3}$

(or $\alpha\bar{\alpha} = 3$)

and $\alpha + \bar{\alpha} = 5$

$\alpha - \bar{\alpha} = \sqrt{13}$

and $\alpha > 0$, $\bar{\alpha} > 0$

This is all just short-hand to make the arithmetic easier

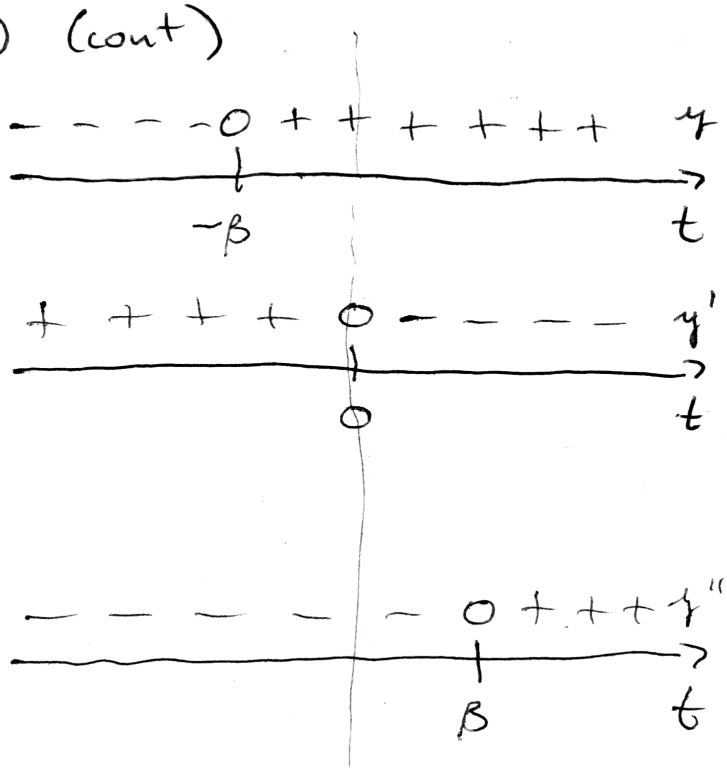
$$1 = y(0) = C + D$$

$$0 = y'(0) = -\alpha C - \bar{\alpha} D$$

$$C = -\frac{\bar{\alpha}}{\alpha - \bar{\alpha}} \qquad D = \frac{\alpha}{\alpha - \bar{\alpha}}$$

NB: $C < 0 \quad D > 0$

(13) (cont)



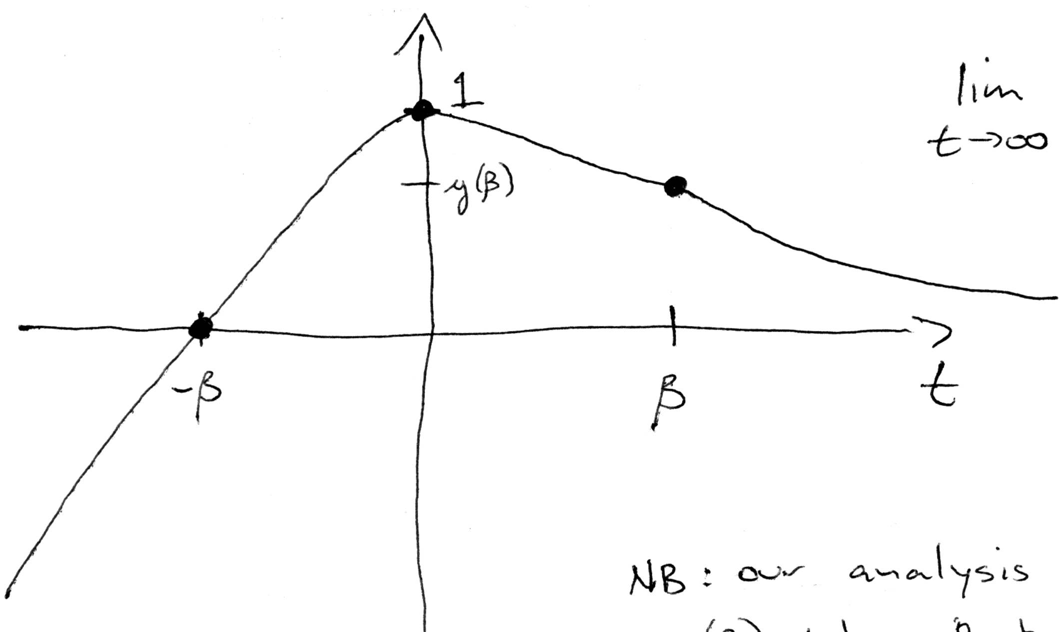

$$y = Ce^{-\alpha t} + De^{-\bar{\alpha} t}$$

$$y' = -\alpha Ce^{-\alpha t} - \bar{\alpha} De^{-\bar{\alpha} t}$$

$$= \frac{3}{\sqrt{13}} e^{-\alpha t} - \frac{3}{\sqrt{13}} e^{-\bar{\alpha} t}$$

$$y'' = -\frac{3\alpha}{\sqrt{13}} e^{-\alpha t} + \frac{3\bar{\alpha}}{\sqrt{13}} e^{-\bar{\alpha} t}$$

Let $\beta = \frac{1}{\sqrt{13}} \log\left(\alpha^2/3\right) > 0$




$$\lim_{t \to \infty} y = 0$$

NB: our analysis proves $y(\beta) < 1$ <u>without</u> approximating.
We can use an approximation to <u>check</u>: $y(\beta) \approx 0.8173 < 1$
We also know $y(-\beta) = 0$ <u>exactly</u>.