§3.2

③ $$W(e^{-2t}, te^{-2t}) = \begin{vmatrix} e^{-2t} & te^{-2t} \\ -2e^{-2t} & e^{-2t} - 2te^{-2t} \end{vmatrix}$$

$$= e^{-2t}\left(e^{-2t} \cancel{- 2te^{-2t}}\right) \cancel{- (-2e^{-2t})(te^{-2t})}$$

$$= e^{-4t}$$

⑤ $$W(e^t \sin t, e^t \cos t) = \begin{vmatrix} e^t \sin t & e^t \cos t \\ e^t \sin t + e^t \cos t & e^t \cos t - e^t \sin t \end{vmatrix}$$

$$= (e^t \sin t)(\cancel{e^t \cos t} - e^t \sin t) - (e^t \cos t)(\cancel{e^t \sin t} + e^t \cos t)$$

$$= (e^{2t})(-\sin^2 t - \cos^2 t)$$

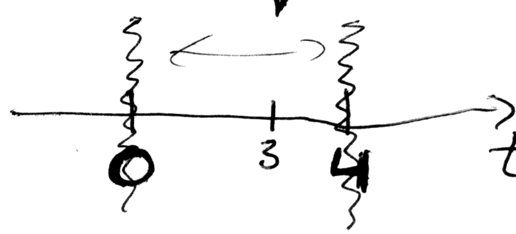
$$= -e^{2t}$$

⑨ $$t(t-4)y'' + 3ty' + 4y = 2$$

$$y(3) = 0 \quad y'(3) = -1$$

$t(t-4) = 0$ @ $t = 0, 4$

$\left.\begin{matrix} 3t \\ 4 \\ 2 \end{matrix}\right\}$ continuous for all t



$$\boxed{0 < t < 4}$$

(13) $t^2 y'' - 2y = 0 \quad t > 0$

$y_1 = t^2$

$y_1' = 2t$ $\qquad t^2(2) - 2(t^2) = 0$ ✓

$y_1'' = 2$

$y_2 = t^{-1}$

$y_2' = -t^{-2}$ $\qquad t^2(2t^{-3}) - 2t^{-1} = 0$ ✓

$y_2'' = 2t^{-3}$

By Thm 3.2.2 (superposition principle), we know $C_1 t^2 + C_2 t^{-1}$ is also a sol'n

(Alt: sub $C_1 t^2 + C_2 t^{-1}$ into the DE & check that it's satisfied.)

(23) $y'' + 4y = 0$

$$\left.\begin{aligned} y_1 &= \cos 2t \\ y_1' &= -2\sin 2t \\ y_1'' &= -4\cos 2t \end{aligned}\right\} \quad -4\cos 2t + 4\cos 2t = 0 \checkmark$$

$$\left.\begin{aligned} y_2 &= \sin 2t \\ y_2' &= 2\cos 2t \\ y_2'' &= -4\sin 2t \end{aligned}\right\} \quad -4\sin 2t + 4\sin 2t = 0 \checkmark$$

(23) (cont)

$$W(y_1, y_2) = \begin{vmatrix} \cos 2t & \sin 2t \\ -2\sin 2t & 2\cos 2t \end{vmatrix}$$

$$= (\cos 2t)(2\cos 2t) - (\sin 2t)(-2\sin 2t)$$

$$= 2\left[\cos^2(2t) + \sin^2(2t)\right] = 2$$

Since $W(y_1, y_2) \neq 0$, yes these are a fundamental solution set

(24) $y'' - 2y' + y = 0$

$y_1 = y_1' = y_2'' = e^t \qquad e^t - 2e^t + e^t = 0$ ✓

$$\left.\begin{aligned} y_2 &= te^t \\ y_2' &= te^t + e^t \\ y_2'' &= te^t + 2e^t \end{aligned}\right\} \quad te^t + 2e^t - 2(te^t + e^t) + te^t = 0 ✓$$

$$W(y_1, y_2) = \begin{vmatrix} e^t & te^t \\ e^t & te^t + e^t \end{vmatrix}$$

$$= e^t(te^t + e^t) - (te^t)(e^t)$$

$$= e^{2t}$$

Since $W(y_1, y_2) \neq 0$, yes these are a fund. sol'n. set

§3.4

(7) $y'' - 2y' + 2y = 0$

$r^2 - 2r + 2 = 0$

$(r-1)^2 + 1 = 0$

$r = 1 \pm i$

$y = Ce^t \cos t + De^t \sin t$

(8) $y'' - 2y' + 6y = 0$

$r^2 - 2r + 6 = 0$

$(r-1)^2 + 5 = 0$

$r = 1 \pm i\sqrt{5}$

$y = Ce^t \cos(\sqrt{5}t) + De^t \sin(\sqrt{5}t)$

(12) $4y'' + 9y = 0$

$4r^2 + 9 = 0$

$r = \pm \frac{3}{2} i$

$y = C \cos\left(\frac{3}{2}t\right) + D \sin\left(\frac{3}{2}t\right)$

(17) $y'' + 4y = 0 \qquad y(0) = 0 \quad y'(0) = 1$

$r^2 + 4 = 0$

$r = \pm 2i$

$y = C\cos(2t) + D\sin(2t)$

$y' = -2C\sin(2t) + 2D\cos(2t)$

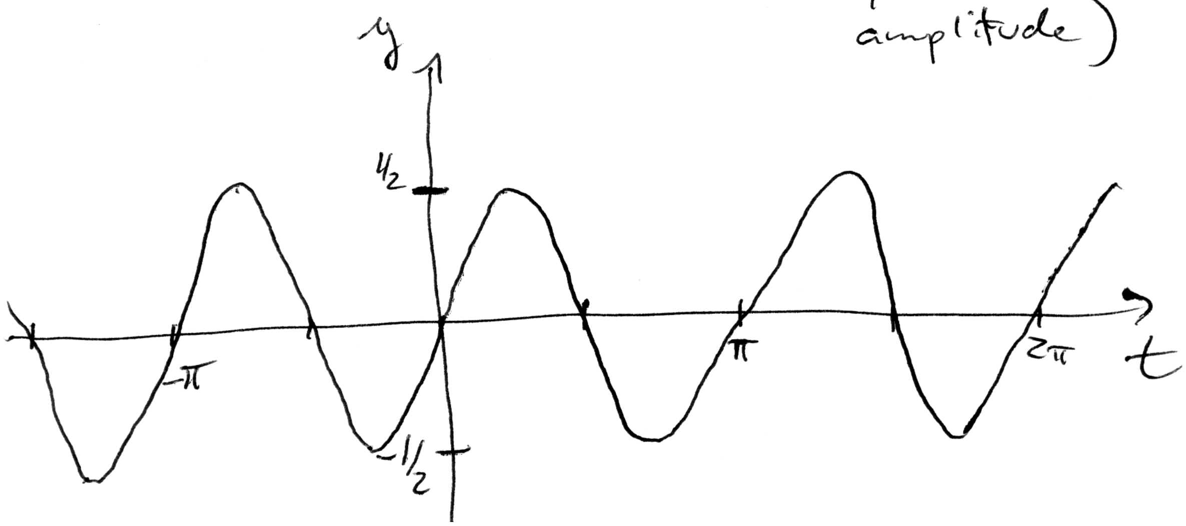
$0 = C \cdot 1 + D \cdot 0$

$1 = -2C \cdot 0 + 2D \cdot 1$

$C = 0 \quad D = 1/2$

$y = \frac{1}{2}\sin(2t)$

$\lim_{t \to \infty} y$ does not exist (y oscillates w/a fixed amplitude)



(19) $y'' - 2y' + 5y = 0 \qquad y(\pi/2) = 0 \qquad y'(\pi/2) = 2$

$$r^2 - 2r + 5 = 0$$

$$(r-1)^2 + 4 = 0$$

$$r = 1 \pm 2i$$

$$y = Ce^t \cos(2t) + De^t \sin(2t)$$

$$y' = C\left(e^t \cos(2t) - 2e^t \sin(2t)\right)$$
$$+ D\left(e^t \sin(2t) + 2e^t \cos(2t)\right)$$

$$0 = Ce^{\pi/2}(-1) + De^{\pi/2} \cdot 0$$

$$2 = C\left(e^{\pi/2}(-1) - 2e^{\pi/2} \cdot 0\right)$$
$$+ D\left(e^{\pi/2} \cdot 0 + 2e^{\pi/2}(-1)\right)$$

$$C = 0 \qquad D = -e^{-\pi/2}$$

$$y = -e^{-\pi/2} e^t \sin(2t)$$

$\lim_{t\to\infty} y$ does not exist (y oscillates with ever increasing amplitude)