(17) (cont)

$$y = C_1 \sin(2t) + C_2 \cos(2t) - \frac{3}{4} t \cos(2t)$$

$$y' = 2C_1 \cos(2t) - 2C_2 \sin(2t) - \frac{3}{4}\cos(2t) + \frac{3}{4} t \cdot 2\sin(2t)$$

$$2 = C_1 \cdot 0 + C_2 \cdot 1 - 0$$

$$-1 = 2C_1 \cdot 1 - 2C_2 \cdot 0 - \frac{3}{4} + 0$$

$$C_1 = -1/8 \qquad C_2 = 2$$

$$y = -\frac{1}{8}\sin(2t) + 2\cos(2t) - \frac{3}{4} t \cos(2t)$$

§3.7

(2) $y'' - y' - 2y = 2e^{-t}$

y_H: $r^2 - r - 2 = 0$

$(r-2)(r+1) = 0$

$$y_H = C_1 e^{2t} + C_2 e^{-t}$$

$$y_I = u_1 e^{2t} + u_2 e^{-t}$$

$$0 = u_1' e^{2t} + u_2' e^{-t}$$

$$2e^{-t} = u_1' 2e^{2t} + u_2' (-e^{-t})$$

$$u_1' = -\frac{e^{-t} \cdot 2e^{-t}}{W(e^{2t}, e^{-t})} \qquad u_2' = \frac{e^{2t} \cdot 2e^{-t}}{W(e^{2t}, e^{-t})}$$

② (cont)

$$w(e^{2t}, e^{-t}) = \begin{vmatrix} e^{2t} & e^{-t} \\ 2e^{2t} & -e^{-t} \end{vmatrix} = -e^{t} - 2e^{t} = -3e^{t}$$

$$u_1 = \int \frac{2}{3} e^{-3t}\, dt = -\frac{2}{9} e^{-3t}$$

$$u_2 = \int -\frac{2}{3}\, dt = -\frac{2}{3} t$$

$$y = C_1 e^{2t} + C_2 e^{-t} \underbrace{- \frac{2}{9} e^{-t} - \frac{2}{3} t e^{-t}}_{\text{one particular sol'n to the inhomog. DE}}$$

NB:

$$y = C_1 e^{2t} + \hat{C}_2 e^{-t} \underbrace{- \frac{2}{3} t e^{-t}}_{\text{another particular sol'n}}$$

Check: (Using Undet. Coeffs)

$$y_H = C_1 e^{2t} + C_2 e^{-t} \qquad \text{RHS} = 2e^{-t}$$

$$-2\left(y_I = \left(Ae^{-t}\right)t\right)$$

$$-1\left(y_I' = (A - At)e^{-t}\right)$$

$$+1\left(y_I'' = (-2A + At)e^{-t}\right)$$

$$2e^{-t} = -3Ae^{-t}$$

② (cont)

$$Z = -3A$$

$$A = -2/3$$

$$y_I = -\frac{2}{3} t e^{-t}$$

⑩ $$y'' - 2y' + y = \frac{e^t}{1+t^2}$$

$$y_H: \quad r^2 - 2r + 1 = 0$$

$$(r-1)^2 = 0$$

$$y_H = C_1 e^t + C_2 t e^t$$

$$y_I = u_1 e^t + u_2 t e^t$$

$$0 = u_1' e^t + u_2' t e^t$$

$$\frac{e^t}{1+t^2} = u_1' e^t + u_2' (e^t + t e^t)$$

$$u_1' = -\frac{(te^t)\left(e^t/1+t^2\right)}{W(e^t, te^t)}$$

$$u_2' = +\frac{(e^t)\left(e^t/1+t^2\right)}{W(e^t, te^t)}$$

(10) (cont)

$$w(e^t, te^t) = \begin{vmatrix} e^t & te^t \\ e^t & (t+1)e^t \end{vmatrix} = (t+1)e^{2t} - te^{2t}$$

$$= e^{2t}$$

$$u_1 = \int -\frac{t}{1+t^2}\,dt = -\frac{1}{2}\log(1+t^2)$$

$$u_2 = \int \frac{1}{1+t^2}\,dt = \tan^{-1} t$$

$$y = C_1 e^t + C_2 te^t \quad \frac{1}{2} e^t \log(1+t^2) + te^t \tan^{-1} t$$

(13)

$$\begin{aligned} 2\,(y_H &= C_1 t^2 + C_2 t^{-1}) \\ +\,0\,(y_H' &= 2C_1 t - C_2 t^{-2}) \\ +\,t^2\,(y_H'' &= 2C_1 + 2C_2 t^{-3}) \\ \hline 0 &= 0 + 0 \quad \checkmark \end{aligned}$$

(Check first) 'cuz the problem sez so

$$y_I = u_1 t^2 + u_2 t^{-1}$$

$$0 = u_1' t^2 + u_2' t^{-1}$$

$$\frac{3t^2-1}{t^2} = u_1' 2t + u_2'(-t^{-2})$$

! (std form RHS)

(13) (cont)

$$u_1' = -\frac{t^{-1} \cdot \left(3t^2-1/t^2\right)}{W(t^2, t^{-1})}$$

$$u_2' = +\frac{t^2\left(3t^2-1/t^2\right)}{W(t^2, t^{-1})}$$

$$W(t^2, t^{-1}) = \begin{vmatrix} t^2 & t^{-1} \\ 2t & -t^{-2} \end{vmatrix} = -1-2 = -3$$

$$u_1 = \int \left(t^{-1} - \tfrac{1}{3}t^{-3}\right) dt = \log t + \tfrac{1}{6}t^{-2}$$

$$u_2 = \int \left(-t^2 + \tfrac{1}{3}\right) dt = -\tfrac{1}{3}t^3 + \tfrac{1}{3}t$$

$$y_I = t^2\left(\log t + \tfrac{1}{6}t^{-2}\right) + t^{-1}\left(-\tfrac{1}{3}t^3 + \tfrac{1}{3}t\right)$$

what they're asking for

NB:

$$y_I = t^2 \log t + \tfrac{1}{2} \underbrace{- \tfrac{1}{3}t^2}_{\text{can be rolled into homog. sol'n}}$$

so

$$\tilde{y}_I = t^2 \log t + \tfrac{1}{2}$$ is another particular sol'n

(15)

$$
\begin{aligned}
1 \quad & \left(y_H = C_1(1+t) + C_2 e^t \right) \\
-(1+t) \quad & \left(y_H' = C_1 + C_2 e^t \right) \\
t \quad & \left(y_H'' = 0 + C_2 e^t \right) \\
\hline
& \quad 0 = 0 + 0 \quad \checkmark
\end{aligned}
$$

$$y_I = u_1(1+t) + u_2 e^t$$

$$0 = u_1'(1+t) + u_2' e^t$$

$$te^{2t} = u_1' \cdot 1 + u_2' e^t$$

! (not $t^2 e^{2t}$)

$$u_1' = -\frac{e^t \cdot te^{2t}}{W(1+t, e^t)}$$

$$u_2' = +\frac{(1+t)\,te^{2t}}{W(1+t, e^t)}$$

$$W(1+t, e^t) = \begin{vmatrix} 1+t & e^t \\ 1 & e^t \end{vmatrix} = te^t$$

(15) (cont)

$$u_1 = \int -e^{2t}\,dt = -\frac{1}{2}e^{2t}$$

$$u_2 = \int (1+t)e^{t}\,dt = te^{t}$$

$$y_I = -\frac{1}{2}e^{2t}(1+t) + te^{t}\cdot e^{t}$$

$$= \frac{1}{2}(t-1)e^{2t}$$