§4.2

(11) $y''' - y'' - y' + y = 0$

$r^3 - r^2 - r + 1 = 0 = (r-1)^2(r+1)$

± 1 are both roots: (1 twice)

1	1	−1	−1	1
		1	0	−1
−1	1	0	−1	0
		−1	1	
1	1	−1	0	
		1		
	1	0		

$y = Ce^t + Dte^t + Ee^{-t}$

(14) $y^{(4)} - 4y''' + 4y'' = 0$

$r^4 - 4r^3 + 4r^2 = 0$

$r^2(r-2)^2 = 0$

$y = C + Dt + Ee^{2t} + Fte^{2t}$

§4.3

② $y^{(4)} - y = 3t + \cos t$

$y_H: r^4 - 1 = 0$

$r = \pm 1, \pm i$

$y_H = Ce^t + De^{-t} + E\cos t + F\sin t$

$y_I = At + B$

$y_I' = A$

$y_I'' = y_I''' = y_I^{(4)} = 0$

$0 - (At + B) = 3t$

$-A = 3$

$B = 0$

$y_I = -3t$

$y_{II} = (A\cos t + B\sin t)t$

$y_{II}' = A\cos t + B\sin t - At\sin t + Bt\cos t$

$y_{II}'' = 2B\cos t - 2A\sin t - Bt\sin t - At\cos t$

$y_{II}''' = -3A\cos t - 3B\sin t + At\sin t - Bt\cos t$

$y_{II}^{(4)} = -4B\cos t + 4A\sin t + Bt\sin t + At\cos t$

$-4B\cos t + 4A\sin t = \cos t$

$B = -1/4 \quad A = 0$

$y = y_H + y_I + y_{II} = Ce^t + De^{-t} + E\cos t + F\sin t - \frac{1}{4}t\sin t - 3t$

④ $y''' - y' = 2\sin t$

$y_H: r^3 - r = 0$

$r = 0, \pm 1$

$y_H = C + De^t + Ee^{-t}$

$y_I = A\sin t + B\cos t$

$y_I' = -B\sin t + A\cos t$

$y_I'' = -A\sin t - B\cos t$

$y_I''' = +B\sin t - A\cos t$

$2B\sin t - 2A\cos t = 2\sin t$

$A = 0 \quad B = 1$

$y = y_H + y_I = C + De^t + Ee^{-t} + \cos t$

§5.1

④ $\sum_0^\infty 2^n x^n$

Root test $\lim \sqrt[n]{|2^n x^n|} \; \begin{matrix} > 1 & \text{div.} \\ < 1 & \text{conv.} \end{matrix}$

$= 2|x|$

$\Rightarrow |x| < 1/2$ for conv.

$\Rightarrow$ ROC is $1/2$

(NB: can also use the ratio test, integral test, comparison test w/ geometric series, et cetera.)

(6) $\sum_{1}^{\infty} \frac{(x-x_0)^n}{n}$

Root test: $\lim \sqrt[n]{\left|\frac{(x-x_0)^n}{n}\right|} \begin{array}{ll} >1 & \text{div.} \\ <1 & \text{conv.} \end{array}$

$$= |x-x_0| \cdot \lim \sqrt[n]{1/n}$$

$$= |x-x_0|$$

$\Rightarrow$ $|x-x_0| < 1$ for conv.

$\Rightarrow$ ROC is 1

(21) $\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} = \sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2} x^n$

$$n_{NEW} = n_{OLD} - 2$$

or $n_{NEW} + 2 = n_{OLD}$

(23) $x \sum_{1}^{\infty} n a_n x^{n-1} + \sum_{0}^{\infty} a_n x^n$

$= \sum_{1}^{\infty} n a_n x^n + \sum_{0}^{\infty} a_n x^n$

$= \sum_{1}^{\infty} n a_n x^n + a_0 x^0 + \sum_{1}^{\infty} a_n x^n$

$= a_0 + \sum_{1}^{\infty} (n+1) a_n x^n$

$\left(= \sum_{0}^{\infty} (n+1) a_n x^n \right)$

(24) $$(1-x^2)\sum_{2}^{\infty} n(n-1)a_n x^{n-2}$$

$$= \sum_{2}^{\infty} n(n-1)a_n x^{n-2} - x^2 \sum_{2}^{\infty} n(n-1)a_n x^{n-2}$$

$\downarrow$ see #21 $\qquad\qquad$ $\downarrow$ distrib. law

$$= \sum_{0}^{\infty} (n+2)(n+1)a_{n+2} x^n - \sum_{2}^{\infty} n(n-1)a_n x^n$$

$$= 2a_2 + 6a_3 x + \sum_{2}^{\infty}\left[(n+2)(n+1)a_{n+2} - (n)(n-1)a_n\right]x^n$$

$$\left(= \sum_{0}^{\infty}\left[(n+2)(n+1)a_{n+2} - n(n-1)a_n\right]x^n \right)$$

(26) $$\sum_{1}^{\infty} n a_n x^{n-1} + x\sum_{0}^{\infty} a_n x^n$$

$$= \sum_{1}^{\infty} n a_n x^{n-1} + \sum_{0}^{\infty} a_n x^{n+1}$$

$\left\{ \begin{array}{l} n_{NEW} = n_{OLD} - 1 \\ n_{NEW} + 1 = n_{OLD} \end{array} \right.$

$\left\{ \begin{array}{l} n_{NEW} = n_{OLD} + 1 \\ n_{NEW} - 1 = n_{OLD} \end{array} \right.$

$$= \sum_{0}^{\infty} (n+1)a_{n+1} x^n + \sum_{1}^{\infty} a_{n-1} x^n$$

$$= a_1 + \sum_{0}^{\infty}\left[(n+1)a_n + a_{n-1}\right]x^n$$

$$\left(\neq \sum_{0}^{\infty}\left[(n+1)a_n + a_{n-1}\right]x^n \right)$$

!