p168 (28) $(x-1)y'' - xy' + y = 0 \qquad x > 1$

$y_1(x) = e^x$

2^{ND} order linear homo. w/ one known sol'n $\Rightarrow$ red of order applies

$y_2(x) = v(x)\, y_1(x)$

$$(x-1)(ve^x)'' - x(ve^x)' + ve^x = 0$$

$$(x-1)(v''e^x + 2v'e^x + \cancel{ve^x})$$
$$-x(v'e^x + \cancel{ve^x}) + \cancel{ve^x} = 0$$

Cancel the e^x's too:

$$(x-1)(v'' + 2v') - xv' = 0$$

$$(x-1)(v')' + (x-2)v' = 0$$

$$(v')' + \left(\frac{x-2}{x-1}\right)v' = 0 \qquad (x > 1 \text{ so this is OK})$$

$$IF = \exp\left(\int \frac{x-2}{x-1}\,dx\right)$$
$$= \exp\left(\int \left(1 - \frac{1}{x-1}\right)dx\right)$$
$$= \exp(x - \log(x-1)) = \frac{1}{x-1}e^x$$

$$\Rightarrow v' = C(x-1)e^{-x}$$

$$\Rightarrow v = -C(x-1)e^{-x} - Ce^{-x} + D$$
$$= -Cxe^{-x} + D$$

$$\Rightarrow y_2 = -Cx + De^x$$ (this is actully the gen'l sol'n too)