p136 (10) $y'' + 4y' + 3y = 0 \qquad y(0)=2 \quad y'(0)=-1$

The characteristic equ is:

$$r^2 + 4r + 3 = 0$$

which has roots $r_1 = -1$ & $r_2 = -3$

Thus every sol'n is of the form:

$$y = C_1 e^{-t} + C_2 e^{-3t} \qquad \left(\text{so that } y' = -C_1 e^{-t} - 3C_2 e^{-3t}\right)$$

Since $y(0)=2$,

$$2 = C_1 e^0 + C_2 e^0 = C_1 + C_2$$

Since $y'(0)=-1$

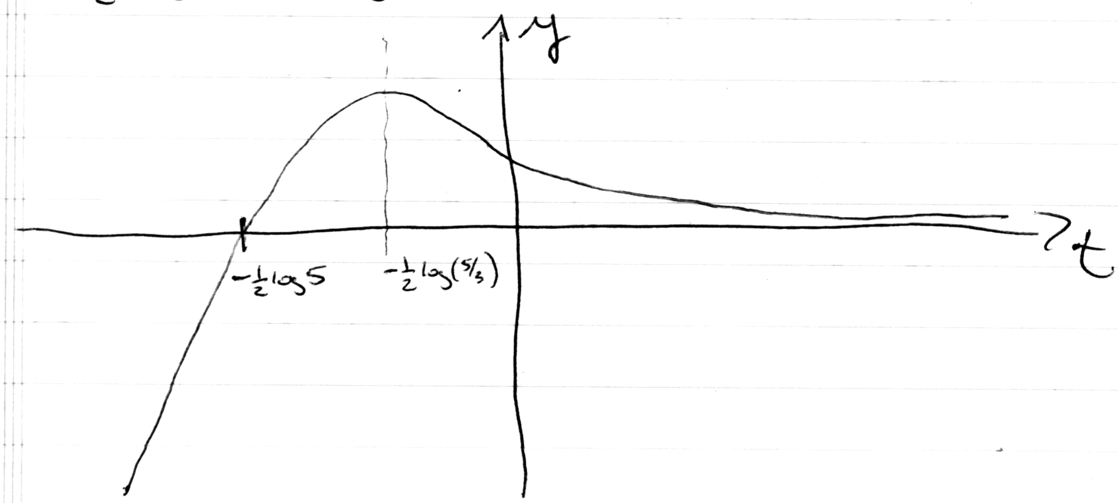
$$-1 = -C_1 - 3C_2$$

Together these imply $C_1 = \frac{5}{2}$ & $C_2 = -\frac{1}{2}$ so that

$$y = \frac{5}{2}e^{-t} - \frac{1}{2}e^{-3t}$$

is the sol'n of our particular IVP

$$\lim_{t\to\infty} y = \lim_{t\to\infty}\left(\frac{5}{2}e^{-t} - \frac{1}{2}e^{-3t}\right) = 0 - 0 = 0$$



p136 21 ⓐ Solve the IVP $y''-y'-2y=0$ $y(0)=\alpha$
$y'(0)=2$

The char eqn is:

$r^2-r-2=0$

It has roots $r_1=-1$ $r_1=2$ so that

$y=C_1e^{-t}+C_2e^{2t}$

$y'=-C_1e^{-t}+2C_2e^{2t}$

$y(0)=\alpha \Rightarrow \alpha=C_1+C_2$

$y'(0)=2 \Rightarrow 2=-C_1+2C_2$

$\Rightarrow C_1=\dfrac{2\alpha-2}{3}$ $C_2=\dfrac{\alpha+2}{3}$

$\Rightarrow$ the sol'n to the IVP is $y=\left(\frac{2\alpha-2}{3}\right)e^{-t}+\left(\frac{\alpha+2}{3}\right)e^{2t}$

ⓑ $\lim_{t\to\infty} e^{-t}=0$ but $\lim_{t\to\infty} e^{2t}=+\infty$

so in order for $\lim_{t\to\infty} y=0$, we need

$\dfrac{\alpha+2}{3}=0$ or $\alpha=-2$ so that $y=-2e^{-t}$

p152 # 15-21

Hint: use $W[y_1,y_2](t)=\exp\left(-\int p(t)dt\right)$.

for a second order linear homogeneous ODE of the form: $y''+p(t)y'+q(t)y=0$

p158 # 1-28

The answers in the back of the book are all correct as stated.

p 145

(7) $ty'' + 3y = t \qquad y(1) = 1 \qquad y'(1) = 2$

$$\Rightarrow y'' + \frac{3}{t} y = 1 \qquad \text{if } t \neq 0$$

The existence & uniqueness Thm tells us a sol'n exists & is unique so long as $3/t$ is continuous in an interval containing the initial time $t_0 = 1$. Since $3/t$ is continuous $\forall t \neq 0$, and since $t_0 > 0$, we are guaranteed a sol'n $\forall t > 0$.

(12) $(x-2)y'' + y' + (x-2)(\tan x)\, y = 0 \qquad y(3) = 1$
$y'(3) = 2$

$$\Rightarrow y'' + \underbrace{\frac{1}{x-2}}_{\substack{\text{continuous} \\ \text{for } x \neq 2}} y' + \underbrace{(\tan x)}_{\substack{\text{continuous} \\ \text{for } x \neq \frac{(2n+1)\pi}{2} \ \forall n \in \mathbb{Z}}} y = 0 \qquad \forall x \neq 2$$

$$\{x \neq 2\} = (-\infty, 2) \cup (2, \infty)$$

$$\{x \neq \tfrac{(2\pi+1)\pi}{2} \ \forall n \in \mathbb{Z}\} = \cdots \cup \left(-\tfrac{3\pi}{2}, -\tfrac{\pi}{2}\right) \cup \left(-\tfrac{\pi}{2}, \tfrac{\pi}{2}\right) \cup \left(\tfrac{\pi}{2}, \tfrac{3\pi}{2}\right) \cup \cdots$$

and $x_0 = 3 \in (2, \infty)$
$x_0 = 3 \in (\pi/2, 3\pi/2)$

so we ~~the~~ are guaranteed a sol'n for at least all $x \in (2, \infty) \cap (\pi/2, 3\pi/2) = (2, 3\pi/2)$

(13) Hint: The ODE is linear — superposition applies

(14) Hint: The ODE is non-linear — superposition doesn't apply