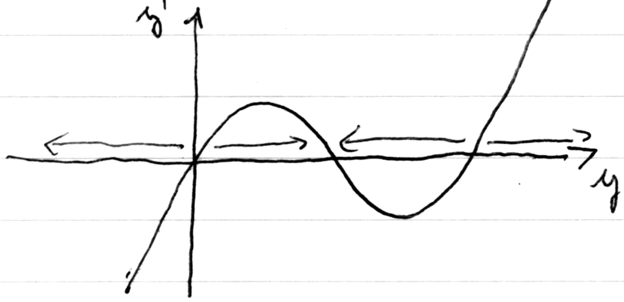
p 84 ③ $y' = y(y-1)(y-2)$

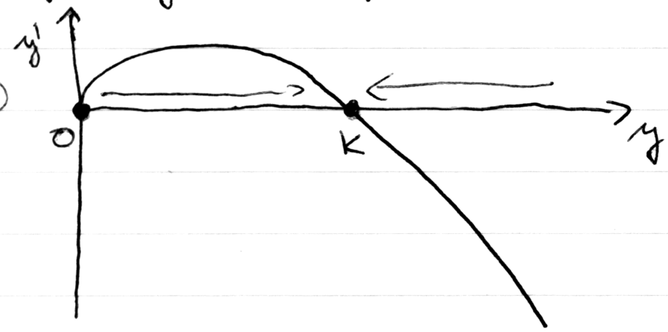


The crit pts are $y=0$ (unstable), $y=1$ (stable), $y=2$ (unstab

$y=0$ (unstable) $\leftrightarrow$ repelling; $y=1$ (stable) $\leftrightarrow$ attractive; $y=2$ (unstab $\leftrightarrow$ repelling

⑯ $y' = ry\log(K/y)$ where $K>0$ & $r>0$

ⓐ



Recall that:
$\lim_{y\to 0^+} y\log y = 0$
$\lim_{y\to 0^+} y\log^2 y = 0$

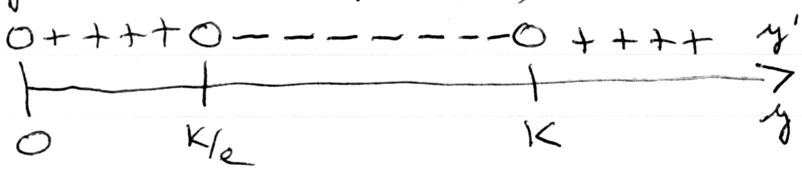
crit pts @ $y=0$ (unstable) & $y=K$ (stable)

ⓑ $y'' = r\left(y'\log(K/y) - y'\right)$

$= ry'\left(\log(K/y) - 1\right)$

$= r^2 y\left(\log(K/y)\right)\left(\log(K/y) - 1\right)$

$y''=0$ when $y = 0, K, K/e$



ⓒ $1 - y/K \le \log(K/y)$ $\forall y>0$ & $K>0$

$\Rightarrow ry(1-y/K) \le ry\log(K/y)$ $\forall r>0$.

Note that $\left(\log(K/y)\right)'' > 0$ and the two agree for $y=K$

p95

② $2x+4y+(2x-2y)y'=0$

$(2x+4y)\,dx+(2x-2y)\,dy=0$

but $\frac{\partial}{\partial y}(2x+4y)=4\neq 2=\frac{\partial}{\partial x}(2x-2y)$

④ $2xy^2+2y+(2x^2y+2x)y'=0$

$(2xy^2+2y)\,dx+(2x^2y+2x)\,dy=0$

$\frac{\partial}{\partial y}(2xy^2+2y)=4xy+2=\frac{\partial}{\partial x}(2x^2y+2)$ ✓

$\Rightarrow \exists \psi$ s.t.

$\frac{\partial\psi}{\partial x}=2xy^2+2y$ & $\frac{\partial\psi}{\partial y}=2x^2y+2x$

$\Downarrow$

$\psi=x^2y^2+2xy+C(y)$

$\Downarrow$

$\frac{\partial\psi}{\partial y}=2x^2y+2x+C'(y)$

$2x^2y+2x+C'(y)=2x^2y+2x$

$C'(y)=0$

$C(y)=\text{const}$

$\Rightarrow$ y def'd by $x^2y^2+2xy=\text{const}$ solves the DE

㉑ $y\,dx+(2x-ye^y)\,dy=0$

$\frac{\partial}{\partial y}(y)=1\neq 2=\frac{\partial}{\partial x}(2x-ye^y)$

multiply by $\mu=y$ $\Rightarrow$ $y^2\,dx+y(2x-ye^y)\,dy=0$

$\frac{\partial}{\partial y}(y^2)=2y=\frac{\partial}{\partial x}\left(y(2x-ye^y)\right)$

$\Rightarrow \exists\psi$ s.t. $\frac{\partial\psi}{\partial x}=y^2$ & $\frac{\partial\psi}{\partial y}=y(2x-ye^y)$

(21) (cont)

$$\frac{\partial \psi}{\partial y} = y(2x - ye^y)$$

$$\Rightarrow \psi = xy^2 - y^2e^y + 2ye^y - 2e^y + C(x)$$

$$\Rightarrow \frac{\partial \psi}{\partial x} = y^2 + C'(x)$$

$$\Rightarrow y^2 + C'(x) = y^2 \quad \Rightarrow C'(x) = 0 \quad \Rightarrow C(x) = \text{const}$$

$$\Rightarrow \underset{y \text{ def'd by}}{} xy^2 - y^2e^y + 2ye^y - 2e^y = \text{const}$$

solves the DE

(It would've been easier if I reversed the roles of x & y above!)

(30) $$\left(4\frac{x^3}{y^2} + \frac{3}{y}\right)dx + \left(3\frac{x}{y^2} + 4y\right)dy = 0$$

$$\frac{\partial}{\partial y}\left(4\frac{x^3}{y^2} + \frac{3}{y}\right) = -8\frac{x^3}{y^3} - \frac{3}{y^2} \neq \frac{3}{y^2} = \frac{\partial}{\partial x}\left(3\frac{x}{y^2} + 4y\right)$$

We want to find μ so that $(\mu F)_y = (\mu G)_x$

where $F = 4x^3y^{-2} + 3y^{-1}$ & $G = 3xy^{-2} + 4y$

$$\Rightarrow \mu_y F + \mu F_y = \mu_x G + \mu G_x$$

S'pose $\mu = \mu(x)$ so $\mu_y = 0$. Then

$$\mu\left(-8x^3y^{-3} - 3y^{-2}\right) = \mu'\left(3xy^{-2} + 4y\right) + \mu \cdot 3y^{-2}$$

$$\Rightarrow \underbrace{\frac{\mu'}{\mu}}_{\text{a pure fcn of } x} = \left(\frac{3xy^{-2} + 4y}{-8x^3y^{-3} - 6y^{-2}}\right)^{-1}$$

$\Rightarrow$ this isn't possible!

$\Rightarrow$ our assumption that $\mu = \mu(x)$ must be incorrect p95 (30) (cont) Try assuming $\mu = \mu(y)$ so that $\mu_x = 0$.

$$\Rightarrow \mu'(4x^3y^{-2} + 3y^{-1}) + \mu(-8x^3y^{-3} - 3y^{-2}) = \mu \cdot 3y^{-2}$$

$$\Rightarrow \mu'(4x^3y^{-2} + 3y^{-1}) = \mu\underbrace{(8x^3y^{-3} + 6y^{-2})}_{2y^{-1}(4x^3y^{-2} + 3y^{-1})}$$

$$\Rightarrow \mu' = \mu \cdot \frac{2}{y} \Rightarrow \mu = Cy^2$$

We free to pick C; we might as well let $C=1$.

Multiplying the original eqn by $\mu = y^2$ gives:

$$(4x^3 + 3y)\,dx + (3x + 4y^3)\,dy = 0$$

$$\frac{\partial}{\partial y}(4x^3+3y) = 3 = \frac{\partial}{\partial x}(3x+4y^3)$$ so this indeed is now exact

$$\Rightarrow \exists \psi \text{ s.t. } \psi_x = 4x^3 + 3y \ \& \ \psi_y = 3x + 4y^3$$

$$\psi = x^4 + 3yx + C(y)$$

$$\psi_y = 3x + C'(y)$$

$$3x + C'(y) = 3x + 4y^3$$

$$\Rightarrow C'(y) = 4y^3$$

$$\Rightarrow C(y) = y^4 + \text{const}$$

$\Rightarrow$ y def'd by $x^4 + 3xy + y^4 = \text{const}$

solves the original DE