Solving for y in:

$$\underbrace{y''+2y'+y}_{\substack{L[y] \\ \text{every}}} = \underbrace{3e^{-t}}_{\substack{L[y_1] \\ \text{one}}} + \underbrace{2e^{-t}\cos t}_{\substack{L[y_2] \\ \text{one}}} + \underbrace{4e^{-t}t^2\sin t}_{\substack{L[y_3] \\ \text{one}}} + \underbrace{0}_{\substack{L[y_H] \\ \text{every}}}$$

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y_H : $r^2+2r+1=0$

$$\Rightarrow \quad y_H = C_1e^{-t} + C_2te^{-t}$$

$$y_1 = t^2\cdot Ae^{-t}$$

$$y_1' = -y_1 + 2tAe^{-t}$$

$$y_1'' = -y_1' - \bigcirc + \boxed{2Ae^{-t}}$$

$$\underbrace{y_1''+2y_1'+y_1}_{L[y_1]} = \underbrace{-y_1'}_{y_1-\bigcirc} - \bigcirc + \boxed{\sim} - 2y_1 + 2\bigcirc + y_1$$

$$\Rightarrow \quad L[y_1] = \boxed{\sim}$$

$$L[y_1] = 3e^{-t}, \qquad \boxed{\sim} = 2Ae^{-t}$$

$$\Rightarrow \quad A = 3/2$$

$$y_2 = (e^{-t})(A\sin t + B\cos t)$$

$$y_2' = -y_2 + (e^{-t})(A\cos t \text{ [redacted] } B\sin t)$$

$$y_2'' = -y_2' - \sim + \boxed{(e^{-t})(-A\sin t \text{ [redacted] } B\cos t)}$$

$$\underbrace{y_2'' + 2y_2' + y_2}_{L[y_2]} = -y_2' - \sim + \boxed{\sim} + 2y_2' + y_2$$

$$= y_2' + y_2 - \sim + \boxed{\sim}$$

$$= -y_2 + \sim + y_2 - \sim + \boxed{\sim}$$

$$L[y_2] = \boxed{\sim}$$

$$\begin{array}{cc} \| & \| \\ 2e^{-t}\cos t & (e^{-t})(-A\sin t \text{ [redacted] } B\cos t) \end{array}$$

$$\Rightarrow -A = 0$$

$$-B = 2$$

$$y_3 = (e^{-t})\left[(At^2 + Bt + C)\sin t + (Dt^2 + Et + F)\cos t\right]$$

$$y_3' = -y_3 + (e^{-t})\Big[(2At + B)\sin t + (At^2 + Bt + C)\cos t +$$

$$(2Dt + E)\cos t - (Dt^2 + Et + F)\sin t\Big]$$

$$= -y_3 + (e^{-t})\Big[(-Dt^2 + (2A - E)t + B - F)\sin t +$$

$$(At^2 + (B + 2D)t + C + E)\cos t\Big]$$

$$y_3'' = -y_3' - \sim + (e^{-t})\Big[(-2Dt + 2A - E)\sin t$$

$$+ (-Dt^2 + (2A-E)t + B - F)\cos t + (2At + B + 2D)\cos t$$

$$- (At^2 + (B+2D)t + C + E)\sin t\Big]$$

$$= -y_3' - \sim + (e^{-t})\Big[(-At^2 + (-B-4D)t - C - 2\overset{+2A}{E})\sin t +$$

$$(-Dt^2 + (4A-E)t + 2B + 2D - F)\cos t\Big]$$

$$y_3'' + 2y_3' + y_3 = -y_3' - \sim + \boxed{\sim} + 2y_3' + y_3$$

$$= y_3' + y_3 - \sim + \boxed{\sim}$$

$$= -y_3 + \sim + y_3 - \sim + \boxed{\sim}$$

$$L[y_3] = \boxed{\sim}$$

$$4e^{-t}t^2\sin t \qquad (e^{-t})\Big[(-At^2 + (-B-4D)t - C - 2E + 2A)\sin t$$

$$+ (-Dt^2 + (4A-E)t + 2B + 2D + F)\cos t\Big]$$

$\Rightarrow$ $-A = 4$	$t^2\sin t$		$A = -4$
$-B - 4D = 0$	$t\sin t$		$D = 0$
$-C - 2E + 2A = 0$	$\sin t$	$\Rightarrow$	$E = -16$
$-D = 0$	$t^2\cos t$		$B = 0$
$4A - E = 0$	$t\cos t$		$C = 24$
$2B + 2D + F = 0$	$\cos t$		$F = 0$

Thus

$$y = C_1 e^{-t} + C_2 t e^{-t} +$$

$$\tfrac{3}{2} t^2 e^{-t} +$$

$$(-2) e^{-t} \cos t +$$

$$(e^{-t}) \left[(-4t^2 + 24) \sin t + (-16t) \cos t \right]$$

This problem, in retrospect, has more cancellation with y_i, $\sim$, and $\boxed{\sim}$ than #21, but the ideas still work. Use symbols to make the bookkeeping easier.

It also helps to check. The answer in the back of the book is correct for #21. A computer may help.