Linear & Inhomog. ODEs:

$y = y_H + y_I$

every every one

$\ddot{y} + \dot{y} + y = RHS$

stuff in t

Method of Undetermined Coeffs:

Requires:

① constant coeffs in the DE

② RHS of a special form
(sums & products of exponentials, sines, cosines & polynomials)

Process:

① Make a guess for y_I that "looks like" the RHS (accounting for y_H too)

② Sub. the guess in the DE

③ Match coeffs of like terms & solve for unknown coeffs

④ Assemble sol'n $y = y_H + y_I$

Variation of Parameters

Any coeffs in the DE

Any RHS ← (NB: write in standard form! Divide by ☺)

but...

You must know (or know how to find) y_H already.

S'pose (constants)

$$y_H = C_1 y_1 + C_2 y_2$$

then

$$y_I = u_1 y_1 + u_2 y_2$$

(u_1, u_2: fcns (to be determined))

Sub y_I in the DE to get (w/ simplification)

$$0 = u_1' y_1 + u_2' y_2$$

$$RHS = u_1' y_1' + u_2' y_2'$$

Solving for u_1' & u_2' gives:

$$u_1' = -\frac{y_2 \cdot RHS}{W(y_1, y_2)} \qquad u_2' = \frac{y_1 \cdot RHS}{W(y_1, y_2)}$$

Integrated

Assemble the sol'n $y = y_H + y_I$