$$(x^2-2x-3)y'' + xy' + 4y = 0 = \sum_{n=0}^{\infty} 0 \cdot x^n$$

$$y = \sum_{0}^{\infty} a_n (x-x_0)^n$$

S'pose $x_0 = 0$ 'cuz we're interested in the sol'n near there.

$$y' = \sum_{1}^{\infty} (a_n)(n)(x-x_0)^{n-1}$$

$x_0 = 0$ is a regular point: $0^2 - 2 \cdot 0 - 3 = -3 \neq 0$ and 0 & 4 are finite

$$y'' = \sum_{2}^{\infty} (a_n)(n)(n-1)(x-x_0)^{n-2}$$

$$(x^2-2x-3)y'' = x^2 y'' - 2x y'' - 3y''$$

$$x^2 y'' = x^2 \sum_{n=2}^{\infty} (a_n)(n)(n-1) x^{n-2} = \sum_{n=2}^{\infty} (a_n)(n)(n-1) x^n$$

$$-2xy'' = -2x \sum_{n=2}^{\infty} (a_n)(n)(n-1) x^{n-2} = \sum_{n=2}^{\infty} (-2)(a_n)(n)(n-1) x^{n-1}$$

$$= \sum_{m=1}^{\infty} (-2)(a_{m+1})(m+1)(m) x^{m}$$

with $m = n-1$
$m+1 = n$

$$-3y'' = -3 \sum_{n=2}^{\infty} (a_n)(n)(n-1) x^{n-2}$$

$$= \sum_{m=0}^{\infty} (-3)(a_{m+2})(m+2)(m+1) x^{m}$$

with $m = n-2$
$m+1 = n-1$
$m+2 = n$

$$xy' = x\sum_{n=1}^{\infty}(a_n)(n)x^{n-1} = \sum_{n=1}^{\infty}(a_n)(n)x^n$$

$$4y = \sum_{n=0}^{\infty}(4)(a_n)x^n$$

so the ODE becomes w/ subst:

$$\sum_{n=2}^{\infty}(a_n)(n)(n-1)x^n + \sum_{n=1}^{\infty}(-2)(a_{n+1})(n+1)(n)x^n +$$

$$\sum_{n=0}^{\infty}(-3)(a_{n+2})(n+2)(n+1)x^n + \sum_{n=1}^{\infty}(a_n)(n)x^n$$

$$+ \sum_{n=0}^{\infty}(4)(a_n)x^n = \sum_{n=0}^{\infty}0\cdot x^n$$

I want to match corresponding powers of x (e.g. $1, x, x^2, x^3, \dots$) from the LHS w/ the RHS. Writing out the first few terms:

$$2a_2x^2 + 6a_3x^3 + \dots + (-4)a_2x + (-12)a_3x^2 +$$

$$(-24)a_4x^3 + \dots + \boxed{(-6)a_2} + (-18)a_3x + (-36)a_4x^2 +$$

$$(-60)a_5x^3 + \dots + a_1x + 2a_2x^2 + 3a_3x^3 + \dots$$

$$\boxed{+4a_0} + 4a_1x + 4a_2x^2 + 4a_3x^3 + \dots = \boxed{0} + 0x + 0x^2$$

$$+ 0x^3 + \dots$$

They don't all begin in the same place!

$$\sum_{n=2}^{\infty}(a_n)(n)(n-1)x^n + (-4)a_2x + \sum_{n=2}^{\infty}(-2)(a_{n+1})(n+1)(n)x^n +$$

$$(-6)a_2 + (-18)a_3x + \sum_{n=2}^{\infty}(-3)(a_{n+2})(n+2)(n+1)x^n +$$

$$a_1x + \sum_{n=2}^{\infty}(a_n)(n)x^n + 4a_0 + 4a_1x + \sum_{n=2}^{\infty}(4a_n)x^n$$

$$= 0 + 0x + \sum_{n=2}^{\infty}0 \cdot x^n$$

So:

$$\left[-6a_2 + 4a_0\right] + \left[-4a_2 - 18a_3 + a_1 + 4a_1\right]x +$$

$$\sum_{n=2}^{\infty}\Big[(a_n)(n)(n-1) - 2(a_{n+1})(n+1)(n) - 3(a_{n+2})(n+2)(n+1)$$
$$+ (a_n)(n) + 4a_n\Big]x^n = 0 + 0x + \sum_{n=2}^{\infty}0x^n$$

Thus

$$-6a_2 + 4a_0 = 0 \qquad (x^0 \text{ term})$$

$$-4a_2 - 18a_3 + 5a_1 = 0 \qquad (x \text{ term})$$

$$(n^2+4)a_n - 2(n^2+n)a_{n+1} - 3(n^2+3n+2)a_{n+2} = 0$$

for all $n \geq 2$ (quadratic terms & higher)

or $$a_{n+2} = \frac{(n^2+4)a_n - 2(n^2+n)a_{n+1}}{3(n^2+3n+2)} \qquad \forall n \geq 2$$

Don't forget $y(x) = \sum_{n=0}^{\infty} a_n x^n$ w/ the above specified an. The radius of convergence of this is at least

$$\min\{|0 - -1|, |0-3|\} = 1$$

"x" and "4" have no poles.

min { distances btw our chosen regular pt & all the singular pts }