p314

(17) $$\frac{s-2}{s^2-4s+3} = \frac{A}{s-3} + \frac{B}{s-1} \quad w/ \quad \begin{array}{l} A = 1/2 \\ B = 1/2 \end{array}$$

$$\mathcal{L}^{-1}\left(\frac{(s-2)e^{-s}}{s^2-4s+3}\right) = \mathcal{L}^{-1}\left(e^{-s}\left(\frac{1}{2}\cdot\frac{1}{s-3} + \frac{1}{2}\frac{1}{s-1}\right)\right)$$

$$= u_1(t)\left(\frac{1}{2}e^{3(t-1)} + \frac{1}{2}e^{(t-1)}\right)$$

$$\mathcal{L}^{-1}\left(e^{-s}\cdot\frac{s-2}{(s-2)^2-1}\right) = u_1(t)\, e^{2(t-1)} \cosh(t-1)$$

(19) (a) We know $F(s) = \mathcal{L}\{f(t)\}(s)$ exists for $s > a$

Also $c > 0$ so $1/c$ makes sense, and

$F(s/c)$ exists for $s/c > a$ or $s > ac$.

OK 'cuz $c > 0$

Thus

$$\frac{1}{c}F\left(\frac{s}{c}\right) = \frac{1}{c}\int_0^\infty f(t)\, e^{-\left(\frac{s}{c}\right)t}\, dt$$

$$= \frac{1}{c}\int_0^\infty f(cu)\, e^{-su}\, c\,du \qquad w/ \begin{array}{l} t = cu \\ dt = c\,du \end{array} \ \& \ c > 0$$

$$= \int_0^\infty f(ct)\, e^{-st}\, dt \qquad \frac{1}{c}\cdot c = 1 \ \& \ u = t$$

$$= \mathcal{L}\{f(ct)\}(s)$$

(19) (b) We know from (a) that for $c>0$

$$\mathcal{L}\{f(ct)\}(s) = \frac{1}{c} F\left(\frac{s}{c}\right)$$

Let $k = \frac{1}{c} > 0$. Then

$$\mathcal{L}\left\{f\left(\frac{t}{k}\right)\right\}(s) = k F(ks)$$

Multiplying by $\frac{1}{k}$ & bringing the $\frac{1}{k}$ inside $\mathcal{L}$:

$$\mathcal{L}\left\{\frac{1}{k} f\left(\frac{t}{k}\right)\right\}(s) = F(ks)$$

Taking $\mathcal{L}^{-1}$ of both sides:

$$\frac{1}{k} f\left(\frac{t}{k}\right) = \mathcal{L}^{-1}\{F(ks)\}(t)$$

(c) $$\mathcal{L}\left\{\frac{1}{a} e^{-bt/a} f\left(\frac{t}{a}\right)\right\}(s) = \int_0^\infty \frac{1}{a} e^{-bt/a} f\left(\frac{t}{a}\right) e^{-st}\, dt$$

$u = t/a$
& $a > 0$

$$= \int_0^\infty e^{-bu} f(u)\, e^{-(as)u}\, du$$

$$= \int_0^\infty f(u)\, e^{-(as+b)u}\, du$$

$$= \mathcal{L}\{f(t)\}(as+b)$$

$$= F(as+b)$$

Thus $\frac{1}{a} e^{-bt/a} f\left(\frac{t}{a}\right) = \mathcal{L}^{-1}\{F(as+b)\}(t)$

p321

② $y'' + 2y' + 2y = u_\pi(t) - u_{2\pi}(t)$

$y(0) = 0$

$y'(0) = 1$

Let $\mathcal{L}(y) = Y$

$$\left(s^2 Y - s\cdot 0 - 1\right) + 2\left(sY - 0\right) + 2Y = \left(e^{-\pi s} - e^{-2\pi s}\right)\frac{1}{s}$$

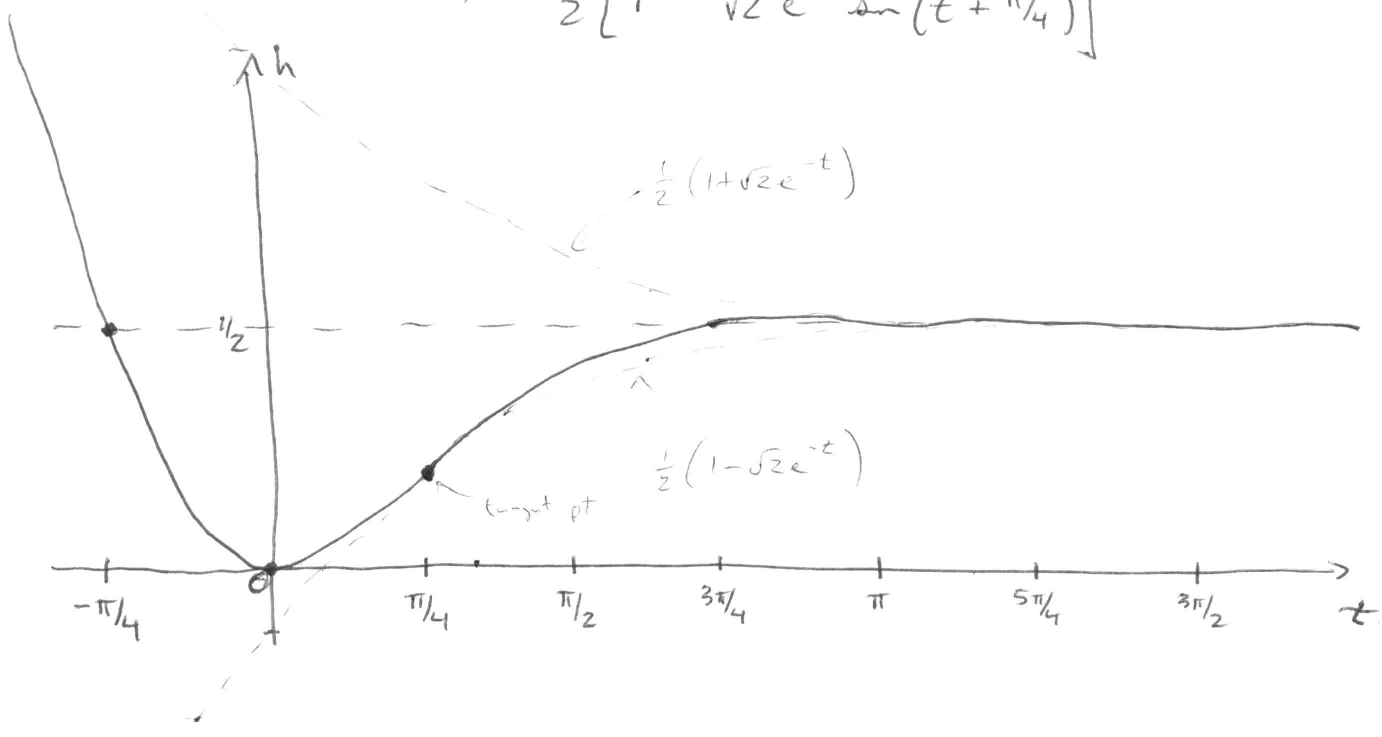
$$\left(s^2 + 2s + 2\right)Y = \left(e^{-\pi s} - e^{-2\pi s}\right)\frac{1}{s} + 1$$

$$\frac{1}{s(s^2+2s+2)} = \frac{A}{s} + \frac{B(s+1)+C}{(s+1)^2+1} \quad w/ \quad A = 1/2,\ C = -1/2,\ B = -1/2$$

$$Y = \left(e^{-\pi s} - e^{-2\pi s}\right)\underbrace{\left(\frac{1}{2}\cdot\frac{1}{s} - \frac{1}{2}\,\frac{1}{(s+1)^2+1^2} - \frac{1}{2}\,\frac{s+1}{(s+1)^2+1^2}\right)}_{H(s)} + \frac{1}{(s+1)^2+1^2}$$

$$\mathcal{L}^{-1}\{H(s)\}(t) = h(t) = \frac{1}{2}\left[1 - e^{-t}\sin t - e^{-t}\cos t\right]$$

$$h(t) = \frac{1}{2}\left[1 - \sqrt{2}\,e^{-t}\sin\left(t + \pi/4\right)\right]$$



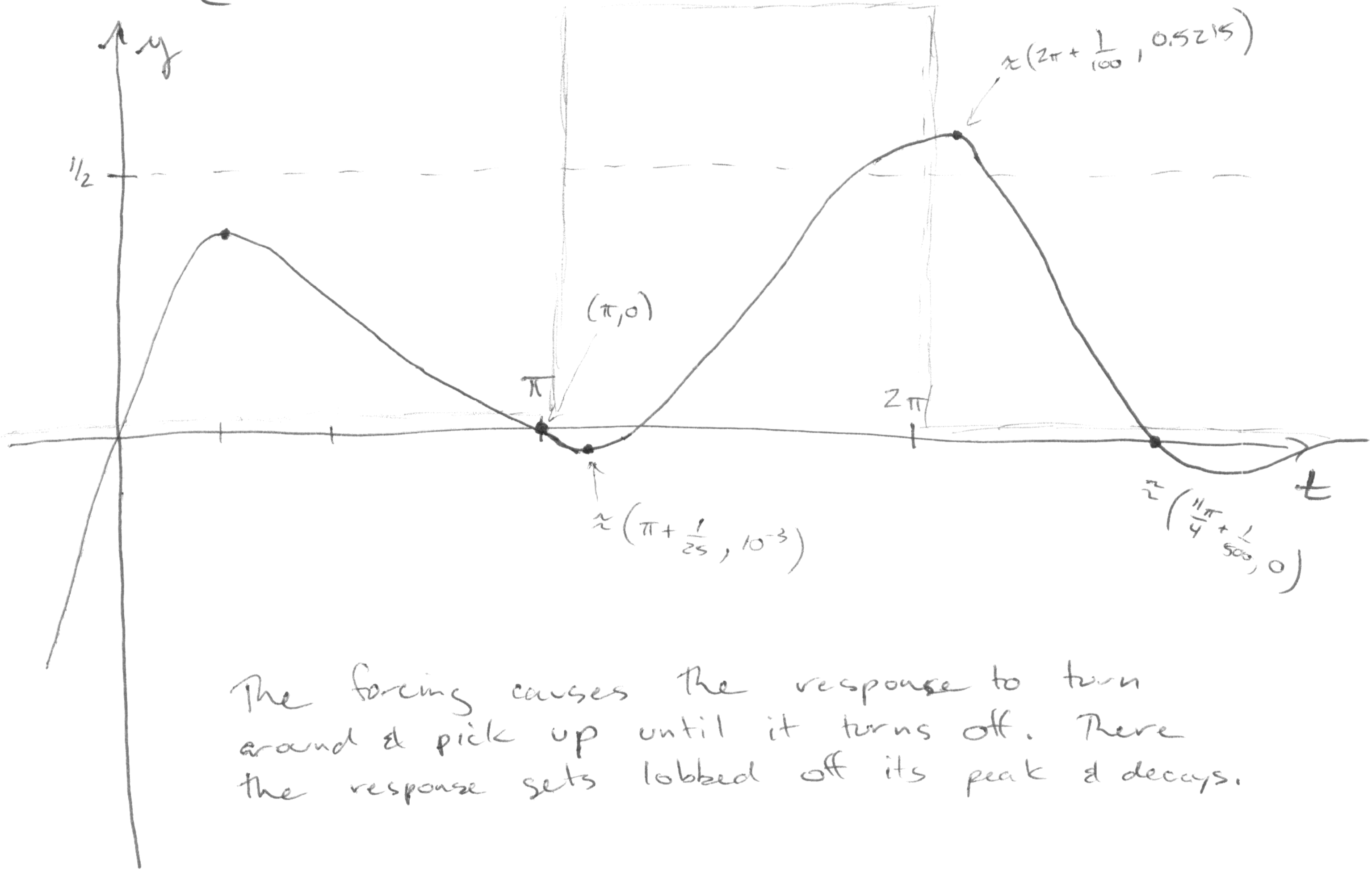
② (cont)

$$Y = \left(e^{-\pi s} - e^{-2\pi s}\right) H(s) + \frac{1}{(s+1)^2+1^2}$$

$$y = \underbrace{u_\pi(t)\, h(t-\pi) - u_{2\pi}(t)\, h(t-2\pi)}_{\substack{\text{from forcing} \\ \text{(w/homog. ICs)}}} + \underbrace{e^{-t}\sin t}_{\substack{\text{from ICs} \\ \text{(non-homog.)}}}$$

(NB: $h(0)=0$ & $h'(0)=0$ so y & y' are continuous. However, $h''(0) \neq 0$ so y'' is not continuous @ $t=\pi, 2\pi$ but is continuous everywhere else.)

$$y = \begin{cases} e^{-t}\sin t & t<\pi \\ h(t-\pi) + e^{-t}\sin t & \pi<t<2\pi \\ h(t-\pi) - h(t-2\pi) + e^{-t}\sin t & t>2\pi \end{cases}$$



The forcing causes the response to turn around & pick up until it turns off. There the response gets lobbed off its peak & decays.

p328

(2) $y'' + 4y = \delta(t-\pi) - \delta(t-2\pi)$

$y(0) = 0 = y'(0)$

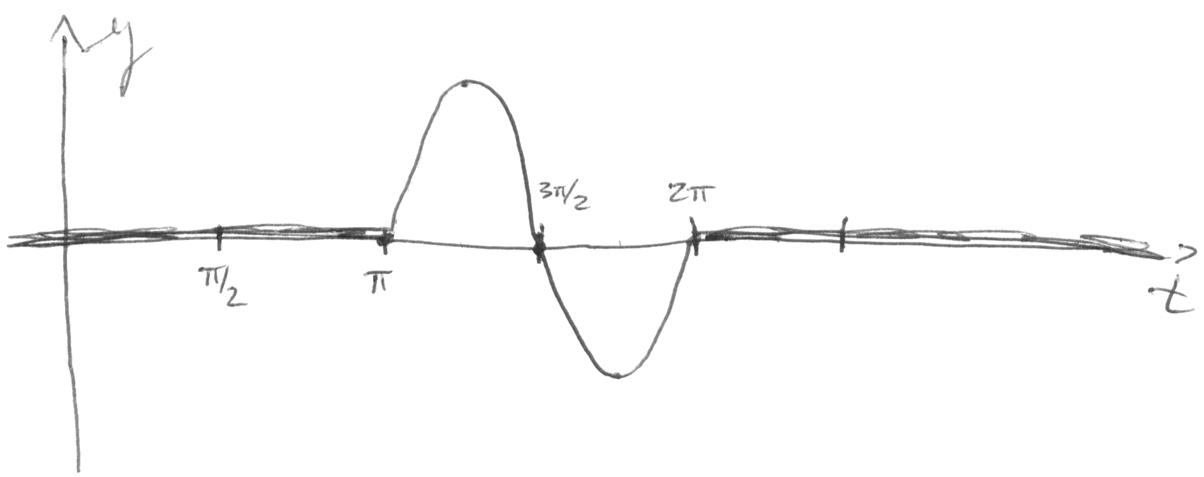
Let $Y = \mathcal{L}(y)$

$$(s^2Y - s\cdot 0 - 0) + 4Y = e^{-\pi s} - e^{-2\pi s}$$

$$Y = \left(e^{-\pi s} - e^{-2\pi s}\right)\frac{1}{s^2+4}$$

$$y = u_\pi(t)\cdot \frac{1}{2}\sin\big(2(t-\pi)\big) - u_{2\pi}(t)\cdot\frac{1}{2}\sin\big(2(t-2\pi)\big)$$

$$y = \big(u_\pi(t) - u_{2\pi}(t)\big)\cdot\frac{1}{2}\sin(2t)$$



pp 330

(17) $y'' + y = \sum_{1}^{20} \delta(t-k\pi)$

$y(0) = 0 = y'(0)$

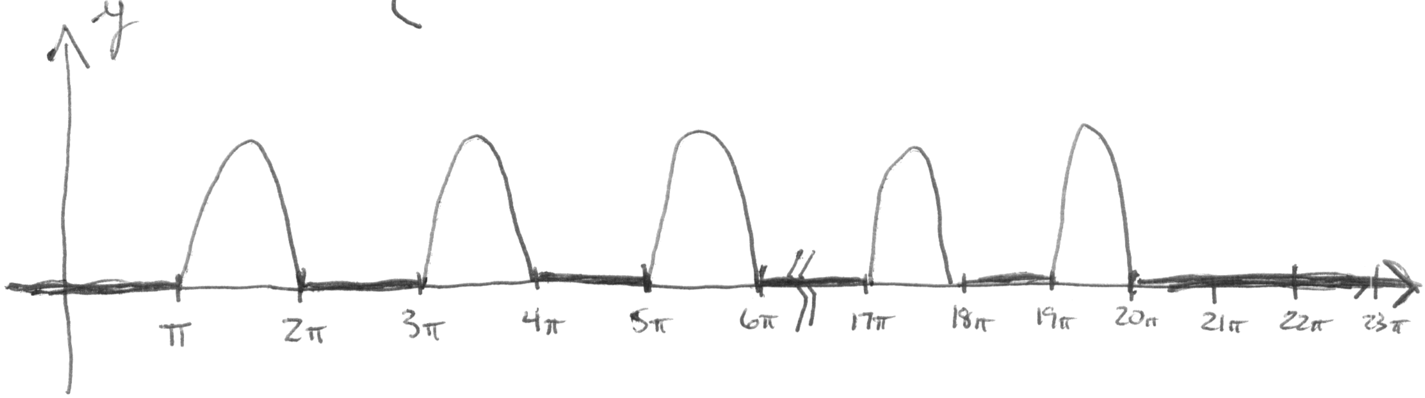
$$(s^2+1)Y = \sum_{1}^{20} \exp(-k\pi s)$$

$$Y = \left(\sum_{1}^{20} \exp(-k\pi s)\right)\cdot \frac{1}{s^2+1^2}$$

$$y = \sum_{1}^{20} u_{k\pi}(t)\,\sin(t-k\pi)$$

$$y = \sum_{1}^{20} u_{k\pi}(t)\cdot(-1)^k \sin(t) = \left(\sum_{1}^{10}\left(-u_{(2n-1)\pi}(t) + u_{2n\pi}(t)\right)\right)\cdot(\sin t)$$

$$y = \begin{cases} 0 & 2n\pi < t < (2n+1)\pi \\ -\sin t & (2n+1)\pi < t < (2n+2)\pi \\ 0 & \text{OW} \end{cases} \quad \text{for any } n,\ 0 \le n \le 9$$



(Obviously $\lim_{t\to\infty} y = 0$)

p 335

(5) $f(t) = \int_0^t e^{-(t-\tau)} \sin\tau \, d\tau$

$= (e^{-t}) * (\sin t)$

so $\mathcal{L}(f(t))(s) = \mathcal{L}(e^{-t})(s) \cdot \mathcal{L}(\sin t)(s)$

$= \frac{1}{s+1} \cdot \frac{1}{s^2+1}$

(17) $y'' + 3y' + 2y = \cos(\alpha t)$

$y(0) = 1 \quad y'(0) = 0$

$(s^2 Y - s \cdot 1 - 0) + 3(sY - 1) + 2Y = \frac{s}{s^2+\alpha^2}$

$(s^2 + 3s + 2)Y = \frac{s}{s^2+\alpha^2} + \underbrace{s+3}_{s+2+1}$

$Y = \frac{s}{s^2+\alpha^2} \cdot \frac{1}{s+2} \cdot \frac{1}{s+1} + \frac{1}{s+1} + \frac{1}{s+2} \cdot \frac{1}{s+1}$

$y = \cos(\alpha t) * e^{-2t} * e^{-t} + e^{-t} + e^{-2t} * e^{-t}$

This is probably the laziest form. There are a variety of possible answers here b/c of the open-endedness of the question.