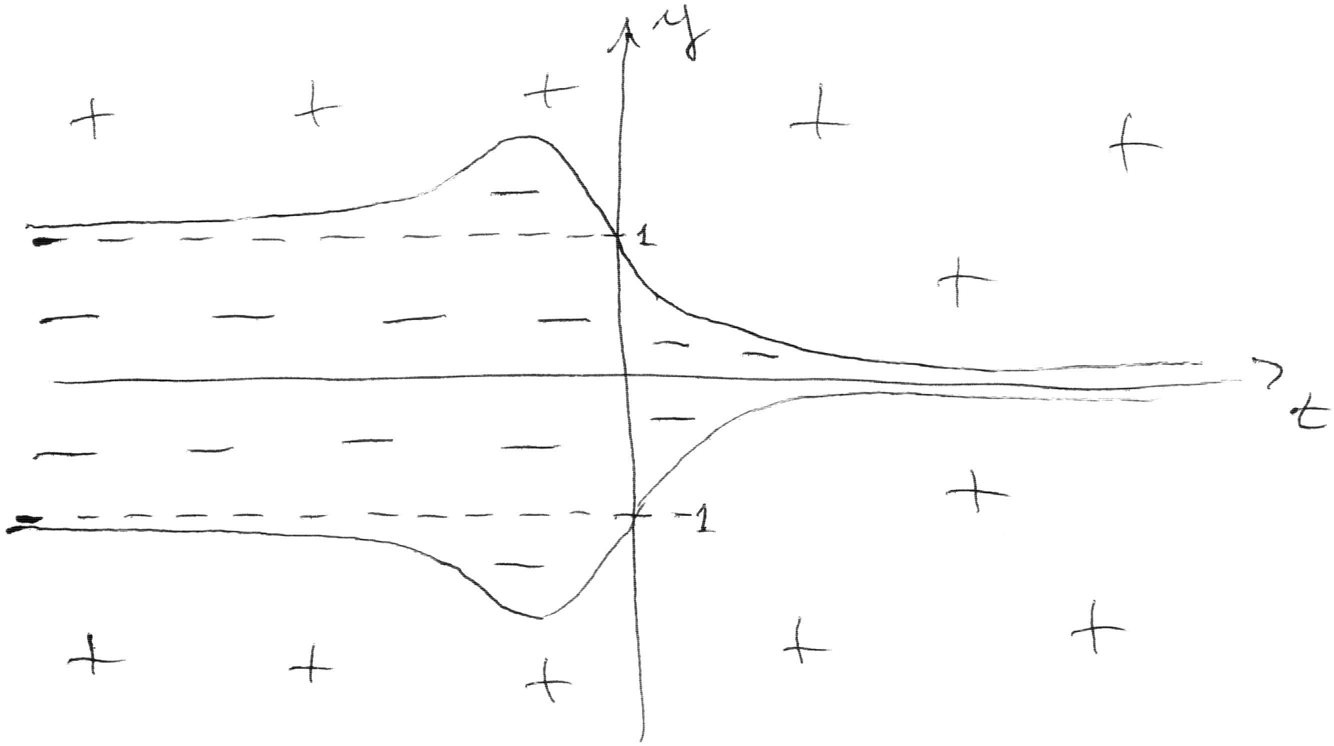
(1) $y' = \log|y| + 2x e^{2x}$

I don't know of any analytic solutions for this ODE. There is, however, quite a bit of analytic information we can still get. For instance, we can plot the sign of y' in the (t, y) plane:



The curves above are of $y = \pm \exp(-2x\exp(2x))$ (NB: They are <u>not</u> sol'n curves). Along these curves, $y' = 0$. The horizontal scale for $t > 0$ has been greatly exaggerated. Note that no solution that starts in the fourth quadrant can ever leave it as t increases.

(2) $y' = 1 - xy$

has sol'ns of the form

$$y = e^{-x^2/2} \int_0^x e^{s^2/2}\, ds + e^{-x^2/2} y_0$$

whenever $y(0) = y_0$

$$\lim_{x \to \infty} y = \underbrace{\lim_{x\to\infty} \left(\frac{\int_0^x e^{s^2/2}\, ds}{e^{x^2/2}} \right)}_{\text{(L'Hopital)}\ \|} + \underbrace{\lim_{x\to\infty} \left(e^{-x^2/2} y_0 \right)}_{\text{always } 0}$$

$$\lim_{x\to\infty} \frac{e^{x^2/2}}{x e^{x^2/2}} = \lim_{x\to\infty} \frac{1}{x} = 0$$

If $y(0) = 0$, $y(4) = e^{-8} \int_0^4 e^{s^2/2}\, ds \approx 0.270396$

Any computer generated sol'ns should be consistent w/ this analysis.

(3) $y' - y = 2t e^{2t}$

$$y = e^t \left(2te^t - 2e^t + 2 + y_0 \right) \quad \text{if } y(0) = y_0$$

$$\lim_{t \to \infty} y = +\infty$$

(4) $y' = \frac{2x}{y(1+x^2)} \Rightarrow y y' = \frac{2x}{1+x^2}$ if $y \neq 0$!

$\Rightarrow y^2/2 = \log|1+x^2| + C$ if $C > 0$, then y is def'd for all x. If $C \leq 0$, then y is def'd only on a limited domain (which may depend on the IC)

(5) $y' = \dfrac{3x^2}{3y^2-4} \Rightarrow (3y^2-4)y' = 3x^2$

only if $y \neq \pm \frac{2}{\sqrt{3}} \approx \pm 1.1547$

$\Rightarrow y^3 - 4y = x^3 + C$

No matter what C is, there will be domain problems w/ the sol'n — sometimes it will be $(-\infty, a)$ or $(b, +\infty)$ or (c,d) w/ $a,b,c,d \in \mathbb{R}$.

$y(1) = 0 \Rightarrow 0 = 1 + C \Rightarrow y^3 - 4y = x^3 - 1$

We want to avoid $y = \pm 2/\sqrt{3}$ and thus

$$\mp \frac{16}{3\sqrt{3}} = x^3 - 1 \Rightarrow x = \left(1 \mp \frac{16}{3\sqrt{3}}\right)^{1/3}$$

≈ -1.27634 & 1.59781

since $-1.27 < \mathbf{1} < 1.59$, then y is only

initial time from IC $y(1)=0$

def'd for $\left(1 - \frac{16}{3\sqrt{3}}\right)^{1/3} < x < \left(1 + \frac{16}{3\sqrt{3}}\right)^{1/3}$