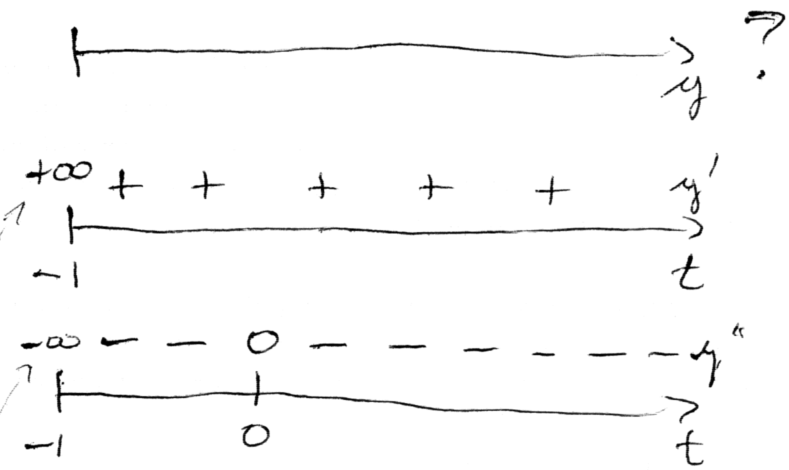
2/2

Graph $y(t) = \int_0^t \frac{ds}{1+s^5}$ for $t > -1$

$y = \int_0^t \frac{ds}{1+s^5}$

$y' = \frac{1}{1+t^5}$

$y'' = -\frac{1}{(1+t^5)^2} \cdot 5t^4$



$\lim_{t \to -1^+} y' = +\infty$ $\qquad$ $\lim_{t \to +\infty} y' = 0$

$\lim_{t \to -1^+} y'' = -\infty$ $\qquad$ $\lim_{t \to +\infty} y'' = 0$

NOTE: (generally, at least)

$\lim_{t \to -1^+} y' = +\infty \;\not\Longrightarrow\; \lim_{t \to -1^+} y = -\infty$

(As an example, think of $\sqrt{t}$ as $t \to 0^+$)

$\lim_{t \to +\infty} y' = 0 \;\not\Longrightarrow\; \lim_{t \to +\infty} y = \text{constant}$

(Though it does imply that y grows more slowly than any polynomial; think $\log t$ as $t \to \infty$)

what we do know so far:

$y(0)=0$ ('cuz $\int_0^0 \frac{ds}{1+s^5} = 0$)

$y' > 0$ always

$y'' \le 0$ always (w/ equality only at $t=0$)

So we can draw:

We also know $\lim_{t\to\infty} y \neq 0$ (that would require $y' < 0$ & an inflection pt @ some $t>0$).

For further analysis:

$$\frac{1}{1+t^5} = \frac{1/5}{1+t} + \underbrace{\frac{\text{some cubic}}{1-t+t^2-t^3+t^4}}_{\text{well behaved near } t=-1}$$

(denominator is near ~~3~~5)
not 0

So

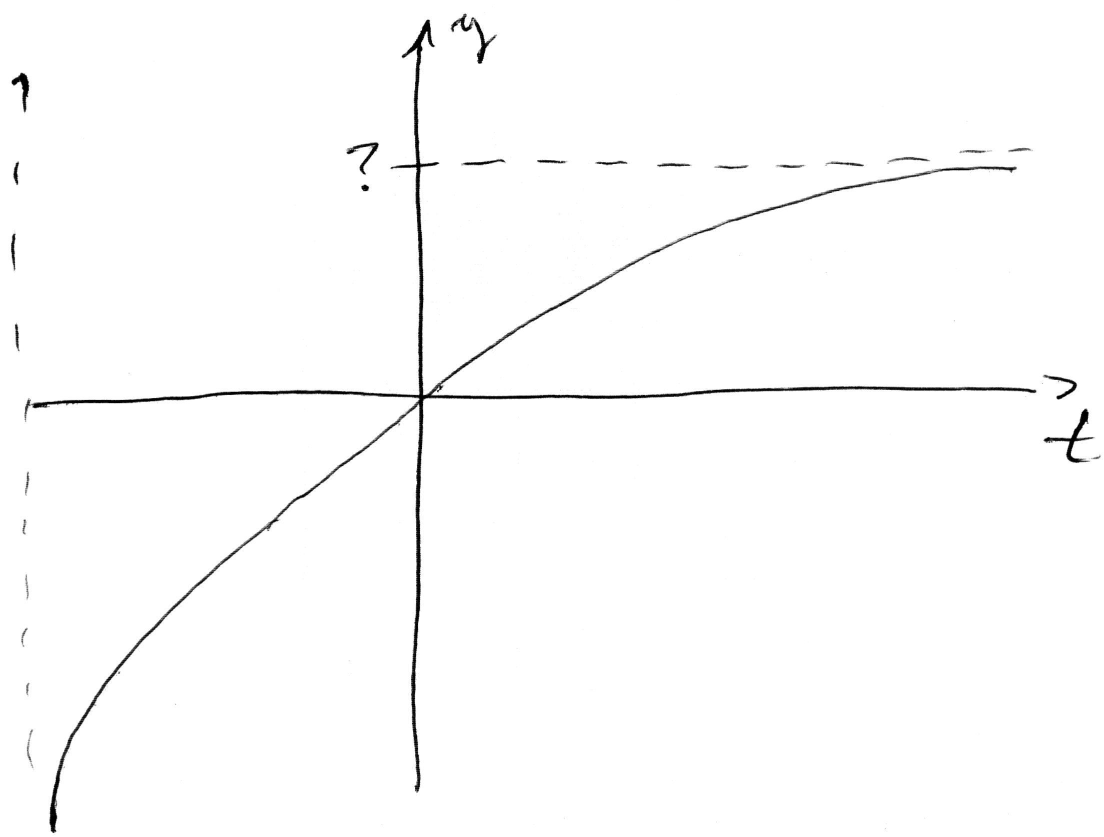
$$\int_0^t \frac{ds}{1+s^5} = \frac{1}{5}\log|1+t| + \text{something well behaved near } t=-1$$

Thus $\lim_{t \to -1^+} y = -\infty$ since $\log|1+t| \to -\infty$

Also, as $t \to +\infty$, $\frac{1}{1+t^5} \longrightarrow \frac{1}{t^5}$

$1/t^5$ has a finite area underneath it & above the t-axis for $t > 1$ (its anti-deriv is $1/t^4$ modulo a constant). Thus $\lim_{t \to +\infty} y = \text{constant} < +\infty$.

Our graph must look like:



Note: we can definitely conclude $y=0$ only for $t=0$.