§3.4

(3) $e^{i\pi} = \cos\pi + i\sin\pi = -1 + 0i = -1$

(8) $y'' - 2y' + 6y = 0$

$r^2 - 2r + 6 = 0$

$(r-1)^2 + 5 = 0$

$r = 1 \pm i\sqrt{5}$

$y = C_1 e^t \cos(\sqrt{5}t) + C_2 \sin(\sqrt{5}t)$

§3.5

(9) $25y'' - 20y' + 4y = 0$

$25r^2 - 20r + 4 = 0$

$(5r-2)^2 = 0$

$r = 2/5, 2/5$

$y = C_1 e^{2/5 t} + C_2 t e^{2/5 t}$

(25) $t^2 y'' + 3ty' + y = 0 \qquad t > 0$

$y_1 = t^{-1}$; Find a sol'n y_2 that is indep of y_1

Method 1:

Def'n of the Wronskian gives

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} t^{-1} & y_2 \\ -t^{-2} & y_2' \end{vmatrix} = t^{-1} y_2' + t^{-2} y_2$$

From Abel's Thm:

$$y'' + \frac{3}{t} y' + \frac{1}{t^2} y = 0$$

$$W = C\exp\left(-\int \frac{3}{t}\, dt\right) = C\exp(-3\log t) = Ct^{-3}$$

Putting the two together:

$$t^{-1} y_2' + t^{-2} y_2 = Ct^{-3}$$

$$y_2' + t^{-1} y_2 = Ct^{-2}$$

$$IF = \exp\left(\int \frac{1}{t}\, dt\right) = t$$

$$(y_2 t)' = Ct^{-1}$$

$$y_2 t = C\log t + D$$

$$y_2 = C\left(\frac{\log t}{t}\right) + D\left(\frac{1}{t}\right)$$

Any C & D will make this y_2 a sol'n.
However we need $C \neq 0$ to have y_2 be independent of y_1 (o/w $y_2 = D(1/t)$, a multiple of y_1).
Since we're otherwise free to set C & D, for simplicity pick $C=1$, $D=0$, and $y_2 = \log t / t$

<u>Method 2:</u>

$$\begin{aligned}
\text{S'pose} \quad & (y_2 = v y_1 = v t^{-1} &&) \; 1 \\
\text{Then} \quad & (y_2' = v't^{-1} - vt^{-2} &&) \; 3t \\
\text{and} \quad & (y_2'' = v''t^{-1} - 2v't^{-2} + 2vt^{-3} &&) \; t^2 \; +
\end{aligned}$$

$$0 = tv'' + v'$$

$$0 = (tv')'$$

$C = tv' \Rightarrow v' = C/t \Rightarrow v = C\log t + D \Rightarrow y_2 = C\left(\frac{\log t}{t}\right) + D/t$ (same as above)