Quiz 6/18

① True or false: $y = e^{6t}$ is a sol'n of

$$y'' - 17y' + 100y = 0$$

Check: $y = e^{6t}$

$y' = 6e^{6t}$

$y'' = 36e^{6t}$

so $(36e^{6t}) - 17(6e^{6t}) + 100(e^{6t}) = 34e^{6t} \neq 0$

$\boxed{\text{FALSE}}$

② True or false: $\left(\frac{d^2y}{dt^2}\right)^3 + y^3 = (\sin t)^3$ is a second order differential equation.

$\frac{d^2y}{dt^2} = y''$ is the highest order derivative in the DE. $\boxed{\text{TRUE}}$ (That y^3 is a third order polynomial isn't what we're looking for.)

③ True or false: $56y' + 17/y = 0$ is a linear DE.

$17/y$ is a non-linear operation on the unknown y.

$\boxed{\text{FALSE}}$

④ Find the gen'l sol'n of $ty' + 2y = e^t/t$. (for $t>0$)

This is a linear first-order DE. The first step is to put it in STD form:

$$y' + (2/t)\,y = e^t/t^2$$

The second step is to find an integrating factor:

$$IF = \exp\left(\int 2/t \, dt\right)$$

$$= \exp(2 \log t)$$

$$= \exp(\log(t^2)) = t^2$$

(NB: in finding the IF, the constant of integration doesn't matter. Why?)

Next we multiply the DE by the IF:

$$t^2 y' + 2t\,y = e^t$$

The LHS of the DE is guaranteed to now be $(y \cdot IF)'$:

$$\frac{d}{dt}(y \cdot t^2) = e^t$$

Now that the derivative is on the "outside", we integrate:

$$\int \frac{d}{dt}(y \cdot t^2)\,dt = \int e^t\,dt$$

$$y \cdot t^2 = e^t + C$$

Last, we solve for y:

$$\boxed{y = e^t/t^2 + C/t^2}$$

(5) Find the sol'n of the IVP

$$\begin{cases} y' = y(y-1) \\ y(0) = 1/2 \end{cases}$$

The DE is separable:

$$\frac{dy}{dt} = y(y-1)$$

First we separate y & dy's from t & dt's:

$$\Rightarrow \left(\frac{dy}{y(y-1)} = dt\right) \quad \text{or} \quad (y=0) \quad \text{or} \quad (y=1)$$

Next, integrate:

$$\int \frac{dy}{y(y-1)} = \int dt$$

$$\int \left(-\frac{1}{y} + \frac{1}{y-1}\right) dy = \int dt$$

These are sol'ns to the DE 'cuz $y' = 0 = 0 \cdot (-1) = 1 \cdot 0$ but they're not sol'ns to the IVP 'cuz $y(0) \neq 1/2$

$$-\log|y| + \log|y-1| = t + C$$

This is an implicit definition of y in terms of t (you tell me t, and I'll solve an eq'n to find $y(t)$). For most separable (and exact) DEs, this is the best we can do. We'll stop here then.

Last, we need to use the IC to find "C": $y = 1/2$ when $t = 0$ so

$$-\log|1/2| + \log|1/2 - 1| = 0 + C$$

$$0 = 0 + C \Rightarrow C = 0$$

The sol'n to our IVP is thus $\boxed{-\log|y| + \log|y-1| = t}$