For the DE $\quad y'' - 2y' - 3y = 8e^{3t}$

What is a homog-ized version? $\quad y_H'' - 2y_H' - 3y_H = 0$

It's char. eqn.? $\quad r^2 - 2r - 3 = 0$

It's roots $\quad r = 3, -1$

Sol'n? $\quad y_H = c_1 e^{3t} + c_2 e^{-t}$

Fund. sol'n set $\quad \{e^{3t}, e^{-t}\}$

Wronskian (two ways)

$$W = \begin{vmatrix} e^{3t} & e^{-t} \\ 3e^{3t} & -e^{-t} \end{vmatrix} = -4e^{2t}$$

$$W = C\exp\left(-\int -2\,dt\right) = Ce^{2t}$$

Find a particular sol'n (two ways)

Undet coeff. guess:

$$\begin{aligned}
-3 \quad & y_I = (A)e^{3t} \cdot t \\
-2 \quad & y_I' = \left((3A)t + A\right)e^{3t} \\
1 \quad & y_I'' = (9At + 6A)e^{3t}
\end{aligned}$$

$$8e^{3t} = 4Ae^{3t} \qquad A = 2$$

$$y_I = 2te^{3t}$$

Var. of Params guess $\quad y_I = u_1 e^{3t} + u_2 e^{-t}$

$$u_1 = -\int \frac{RHS\, y_2}{W(y_1, y_2)}\,dt = -\int \frac{8e^{3t} \cdot e^{-t}}{-4e^{2t}}\,dt = \int 2\,dt = 2t$$

$$u_2 = +\int \frac{RHS\, y_1}{W(y_1, y_2)} = \int \frac{8e^{3t} \cdot e^{3t}}{-4e^{2t}}\,dt = \int -2e^{4t}\,dt = -\frac{1}{2}e^{4t}$$

$$y_I = 2te^{3t} - \frac{1}{2}e^{3t}$$

$y = y_H + y_I$ Gen'l sol'n

two ways, two diff ans., or are they? no, they're the same: $-\frac{1}{2}e^{3t}$ is part of y_H.