

1.8

⑦  $f(x) = \frac{2}{3}x^3 - \frac{1}{2}x^2 - 10x - 1$

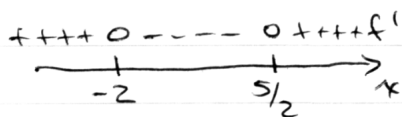
$f$  is defined, continuous, and differentiable everywhere (it is a polynomial). It has no vertical asymptotes. It also has no rational roots ( $f(1), f(-1), f(3), f(-3), f(1/2), f(-1/2), f(3/2), \text{ and } f(-3/2)$  all are not 0; however,  $f$  does have a root between  $x = -6$  &  $x = -3$ , between  $x = -1$  and  $x = 0$ , and between  $x = 3$  and  $x = 6$  — look at the sign of  $f$  and use the Intermediate Value Theorem). As  $x \rightarrow \infty$ ,  $f(x) \rightarrow \infty$  and as  $x \rightarrow -\infty$ ,  $f(x) \rightarrow -\infty$  ( $\frac{2}{3}x^3$  dominates for large  $x$ 's).  $f$  doesn't have any horizontal asymptotes.  $f$  is neither even nor odd, nor periodic.

$$f'(x) = 2x^2 - x - 10$$

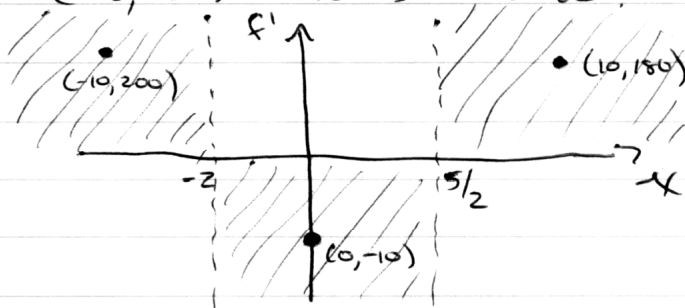
$$= (2x - 5)(x + 2)$$

Also,  $f$  has no cusps.

$f'$  is defined, continuous, and differentiable everywhere. It has no vertical asymptotes. It has roots at  $x = -2$  and  $x = 5/2$  (the rational root theorem succeeds). As  $x \rightarrow \pm\infty$ ,  $f'(x) \rightarrow \infty$ . ( $2x^2$  dominates).  $f'$  doesn't have any horiz. asymptotes and is neither even nor odd nor periodic.  $f'(-2) = 0$  and  $f'(5/2) = 0$ . Since  $f'(-10) = 200$ , then  $f'(x) > 0$  for all  $x \in (-\infty, -2)$ . Since  $f'(0) = -10$ , then  $f'(x) < 0$  for all  $x \in (-2, 5/2)$ . Since  $f'(10) = 180$ , then  $f'(x) > 0$  for all  $x \in (5/2, \infty)$ . To summarize:



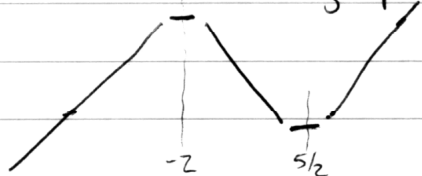
or



① (continued)

Thus  $f$  is increasing on  $(-\infty, -2)$  and  $(5/2, \infty)$  and decreasing on  $(-2, 5/2)$ . There is a local max of  $f$  at  $x = -2$ , and a local min of  $f$  at  $x = 5/2$ .

A quick sketch of the graph of  $f$  right now looks like

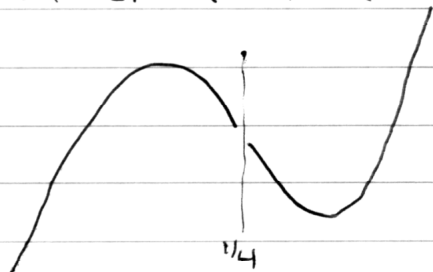


$$f''(x) = 4x - 1$$

$f''$  is a straight line w/ slope 4 and y-intercept at -1.

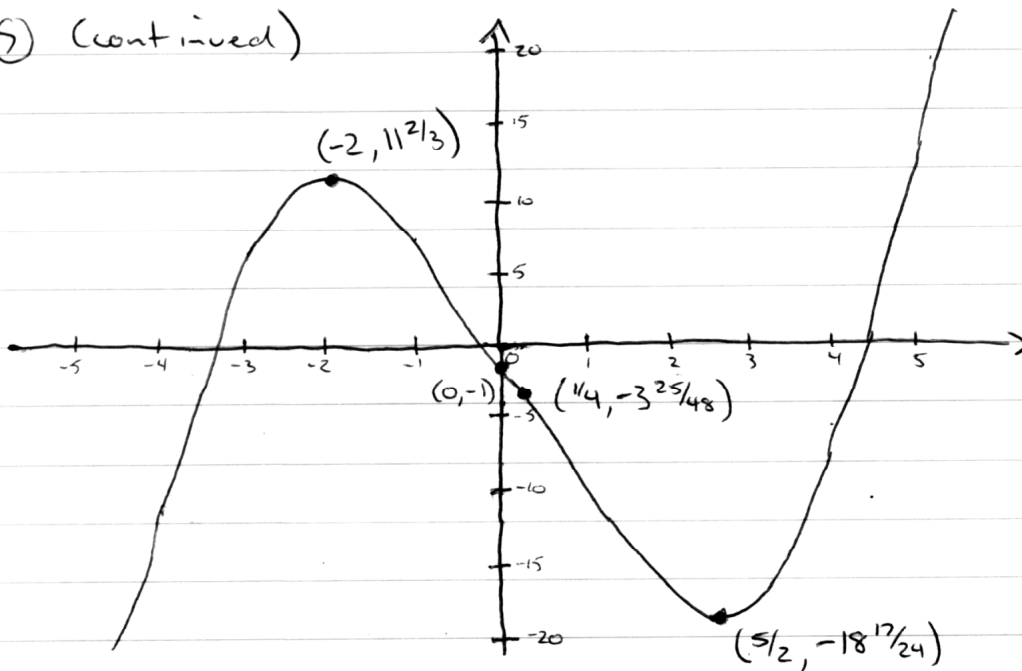
$f''(x) = 0$  for  $x = 1/4$ . For  $x < 1/4$ ,  $f''(x) < 0$ .

For  $x > 1/4$ ,  $f''(x) > 0$ . That is, for  $x < 1/4$  the graph is concave down, and for  $x > 1/4$  the graph is concave up.  $f'$  has a local min at  $x = 1/4$ . A rough sketch of  $f$  from this information looks like:



$x$	$f(x)$	why interesting
$1/4$	$-3\frac{25}{48}$	inflection pt; local min of $f'$ ; change of concavity of $f$
$-2$	$11\frac{2}{3}$	critical pt; local max of $f$
$5/2$	$-18\frac{12}{24}$	critical pt; local min of $f$
$0$	$-1$	y-intercept of $f$
somewhere btw $-1$ & $0$	$0$	x-intercept (zero/root) of $f$
somewhere btw $-6$ & $-3$	$0$	" " " "
somewhere btw $3$ & $6$	$0$	" " " "

⑦ (continued)



④  $f(x) = \frac{1}{4}x - \sqrt{x}$  ~~for  $x \geq 0$~~  ~~for  $x \geq 0$~~  for  $x \geq 0$   
 $= \sqrt{x} \left( \frac{1}{4}\sqrt{x} - 1 \right)$

$f$  is defined and continuous for  $x \geq 0$ .  $f$  is differentiable for  $x > 0$ . (check: use the definitions to verify my statements.)

$f(x) = 0$  for  $x = 0$  and  $x = 16$ . As  $x \rightarrow +\infty$ ,  $f(x) \rightarrow +\infty$  since  $x$  dominates  $\sqrt{x}$ .

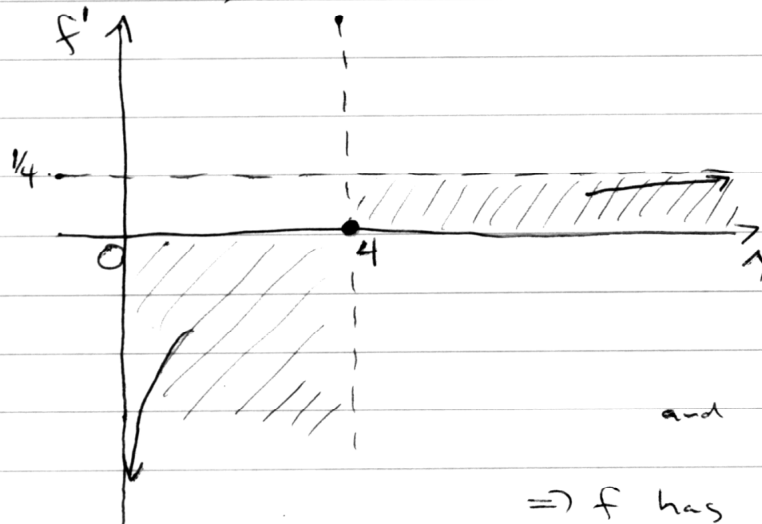


$f(9) = \frac{9}{4} - 3 = -\frac{3}{4} < 0$   $f(25) = \frac{25}{4} - 5 = \frac{5}{4} > 0$

$f'(x) = \frac{1}{4} - \frac{1}{2\sqrt{x}}$  for  $x > 0$

$f'$  is defined, continuous, and differentiable for  $x > 0$ .  $f'$  has a vertical asymptote at  $x = 0$  (as  $x \rightarrow 0^+$ ,  $f'(x) \rightarrow -\infty$ ).  $f'(x) = 0$  for  $x = 4$ . As  $x \rightarrow \infty$ ,  $f'(x) \rightarrow \frac{1}{4}$ ;  $f'$  has a horizontal asymptote at  $y = \frac{1}{4}$ .

⑭ (continued)



$$f'(16) = 1/8 > 0 \text{ so}$$

$$f'(x) > 0 \text{ for } x > 4$$

$$f'(1) = -1/4 < 0 \text{ so}$$

$$f'(x) < 0 \text{ for } 0 < x < 4$$

$$f'(x) < 1/4 \text{ for all } 0 < x$$

$$\lim_{x \rightarrow \infty} f'(x) = 1/4$$

$$\text{and } \lim_{x \rightarrow 0^+} f'(x) = -\infty$$

$\Rightarrow f$  has a local min at  $x = 4$

$$f''(x) = \frac{1}{4x^{3/2}} \text{ for } x > 0$$

$f''$  is def'd, cont., & dif. for  $x > 0$ .  $f''$  has a vertical asymptote at  $x = 0$  (as  $x \rightarrow 0^+$ ,  $f''(x) \rightarrow +\infty$ ).  $f''$  is never 0 (in fact,  $f''(x) > 0$  for all  $x > 0$ ).  $f''$  has ~~vertical~~ a horizontal asymptote at  $y = 0$  (as  $x \rightarrow \infty$  then  $f''(x) \rightarrow 0$ ).

