

Mission: Impossible

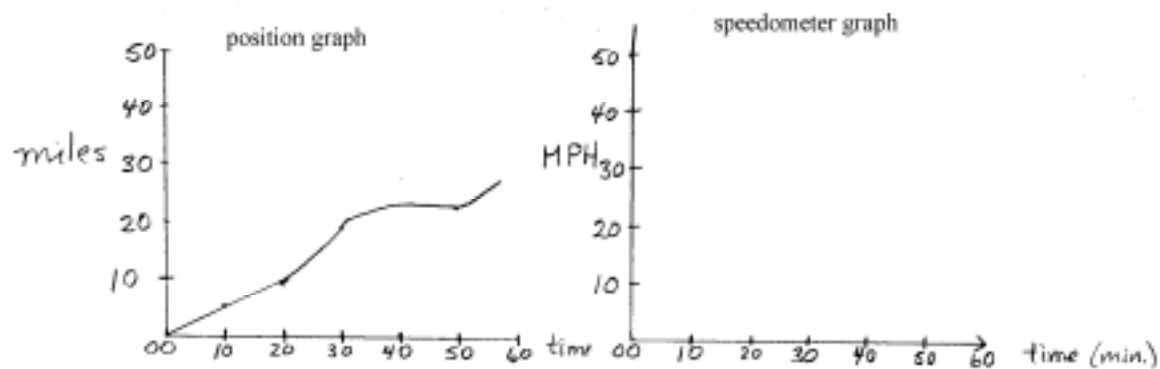
Calculus I (M408C)

Professor Michael Starbird

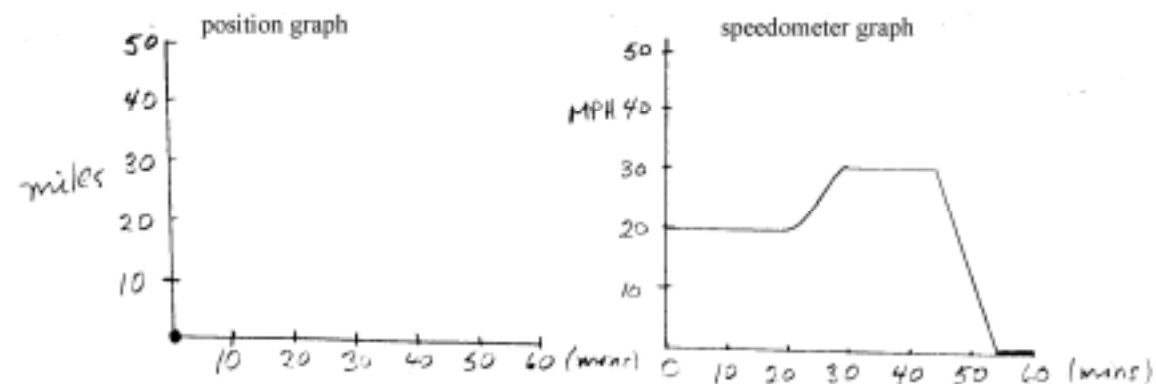
Kidnappers have kidnapped the famous Mission: Impossible celebrity Isaac Newton and are driving along a straight road. Trussed in the back of the van, Sir Isaac has two camcorders: one is taking pictures of the dashboard showing the speedometer (there is no odometer). The camcorder also records the exact time at each instant. The other camcorder sees through a small hole in the side of the van. Visible is an incredibly precise set of roadside mileage markers that mark each nanometer and less and again the camcorder also records the time to the tiniest part of a second.

Question 1: What is a convenient way to display the data gathered by the camcorders? If you think of recording the position moment-by-moment on a graph, you might use minutes and miles rather than hours and miles (since 1 mile per minute is the same as 60 miles per hour).

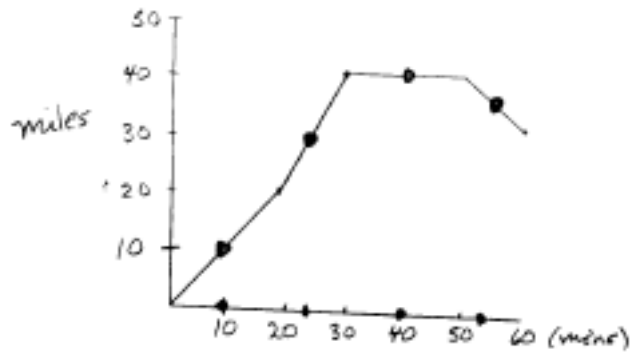
Question 2: During the harrowing ride from 1:00 until 2:00, for some periods of time the speedometer camcorder fails to send the signal (perhaps it is considering self-destructing). If the data on the position camcorder comes through during the times when the speedometer camcorder fails, can the speedometer data be reconstructed? How? Explain. Use the graphs below to help your explanation.



Question 3: During the harrowing ride from 1:00 until 2:00, for some periods of time, the position camcorder fails to send the signal. If the data on the speedometer camcorder comes through during the times when the position camcorder fails, can the position data be reconstructed? How? Explain. Use the graphs below to help your explanation.

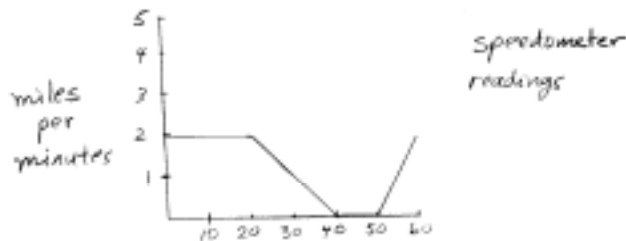


Question 4: If at time t minutes the mileage marker where the van is located is called $p(t)$, how do you figure out the speed of the van at time t in miles per minute? Explain your reasoning and estimate the speedometer reading at each of the times marked on the position graph below.



Question 5: If at time t minutes the mileage marker where the van is located is called $p(t)$, what is the relationship between the speed of the van at time t and the graph of $p(t)$? Explain your reasoning at each of the times marked on the graph above.

Question 6: Suppose at time t the speed of the van is called $v(t)$ miles per minute. Suppose you start at 1:00. How do you figure out how far the van is from the starting point? Use the graph below to illustrate your reasoning. How far has the van gone?



Question 7: Suppose at time t the speed of the van is called $v(t)$. Suppose you start at 1:00. What is the relationship between how far the van is from the starting point and the graph of the function $v(t)$? Use the graph above to illustrate your reasoning.

Suppose $f(x)$ is any function. You could interpret $f(x)$ as telling you the position of a van moving on a straight road at each time x . So you could figure out the velocity of such a van at each time x and also that velocity would bear the relationship to the graph of the curve that you found in Question 5. That velocity function that you find for such a van would then be capable of reconstructing the function $f(x)$ by the process you developed in Question 3 and would have the relationship with the velocity function that you found in Question 7. That reciprocal relationship is the content of The Fundamental Theorem of Calculus. Once you understand this Mission: Impossible scenario, you will understand the main ideas of Calculus I.