

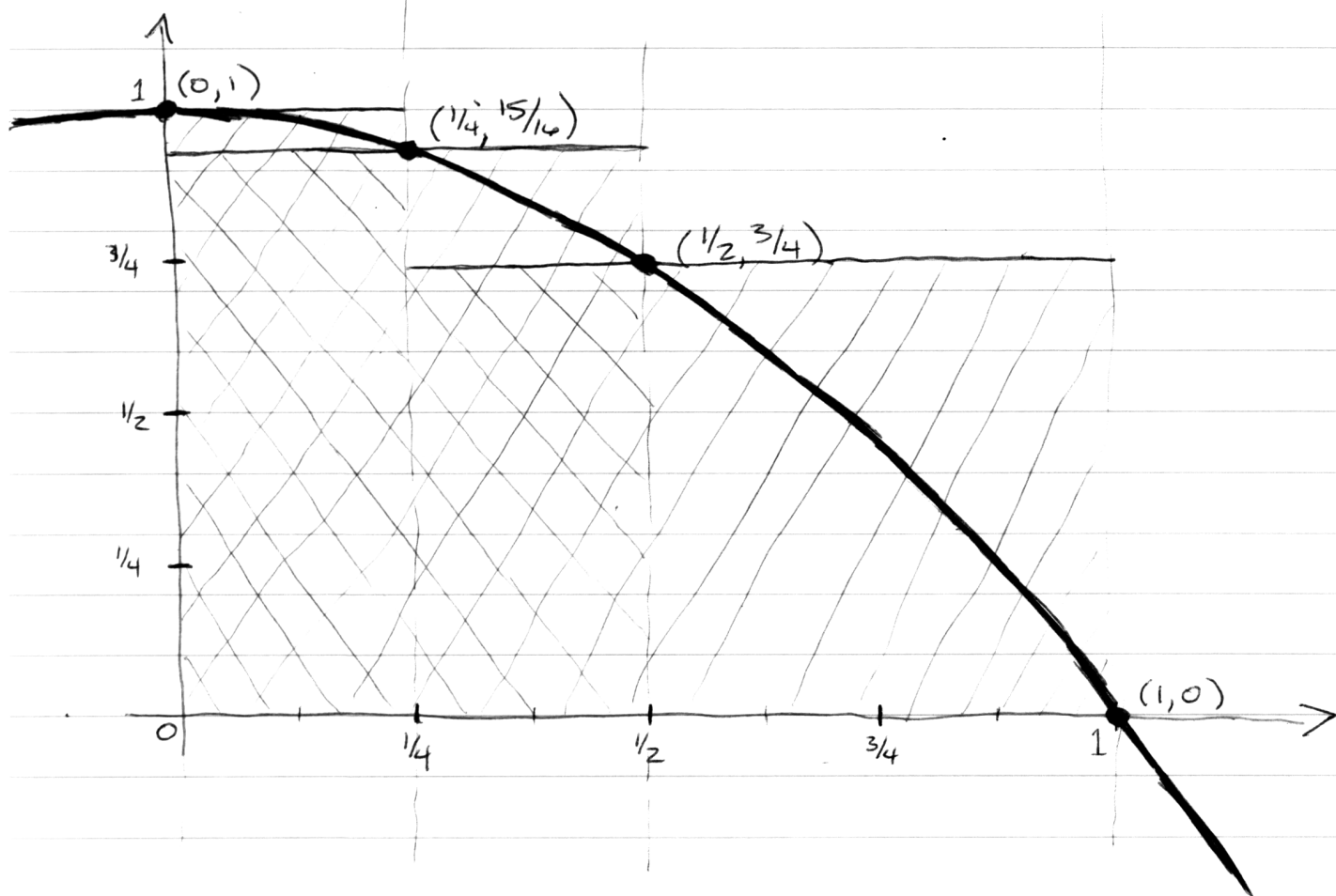
S.1 4

S.2 1, 3, 9, 12, 17

S.3 1, 4, 11, 25, 31

S.1

④ $f(x) = 1 - x^2$ on $[0, 1]$ for $\{0, 1/4, 1/2, 1\}$



$$U_f(P) = (1/4 - 0)(1) + (1/2 - 1/4)(15/16) + (1 - 1/2)(3/4) \\ = 55/64$$

= the area marked by $\parallel\parallel\parallel$ above

$$L_f(P) = (1/4 - 0)(15/16) + (1/2 - 1/4)(3/4) + (1 - 1/2)(0) \\ = 27/64$$

= the area marked by $\parallel\parallel$ above

$$\Rightarrow \frac{27}{64} \leq \int_0^1 (1 - x^2) dx \leq \frac{55}{64}$$

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① Given that $\int_0^1 f(x) dx = 6$, $\int_0^2 f(x) dx = 4$, and $\int_2^5 f(x) dx = 1$

find each of the following:

Ⓐ $\int_0^5 f(x) dx = \int_0^2 f(x) dx + \int_2^5 f(x) dx$

$$= 4 + 1 = 5$$

Ⓑ Since $\int_0^2 f(x) dx = \int_0^1 f(x) dx + \int_1^2 f(x) dx$

then $4 = 6 + \int_1^2 f(x) dx$

and $-2 = \int_1^2 f(x) dx$

Ⓒ $\int_1^5 f(x) dx = \int_1^2 f(x) dx + \int_2^5 f(x) dx$

$$= -2 + 1 = -1$$

↳ from Ⓑ ↳ given

Ⓓ $\int_0^0 f(x) dx = 0$

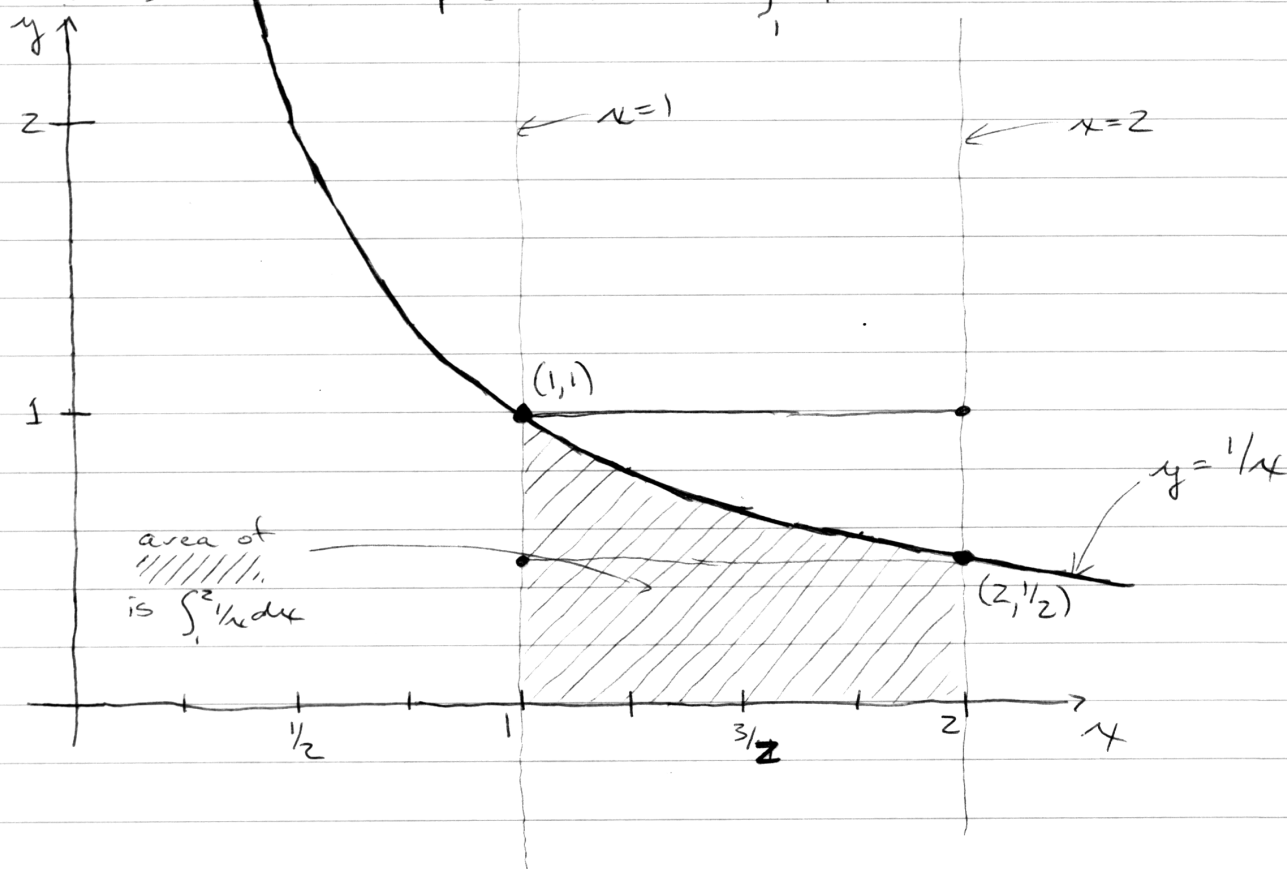
these are "different" 0's
 $\int_{17}^{17} f(x) dx = 0$ also

Ⓔ $\int_2^0 f(x) dx = - \int_0^2 f(x) dx = -(4) = -4$

Ⓕ $\int_5^1 f(x) dx = - \int_1^5 f(x) dx = -(-1) = 1$

③ Use upper & lower sums to show that $\frac{1}{2} < \int_1^2 \frac{1}{x} dx < 1$

Let's draw a picture of $\int_1^2 \frac{1}{x} dx$:



area of box with corners $= 1 \cdot \frac{1}{2} = \frac{1}{2} = L_f(P)$ where
at $(1,0), (2,0), (1, \frac{1}{2}), \& (2, \frac{1}{2})$

$f(x) = 1/x$ and $P = \{1, 2\}$

(the simplest possible set
for a lower sum)
upper or

area of box with corners $= 1 \cdot 1 = 1 = U_f(P)$ where $f(x) = 1/x$
at $(1,0), (2,0), (1,1), \& (2,1)$ and $P = \{1, 2\}$

Thus $L_f(P) = \frac{1}{2} \leq \int_1^2 \frac{1}{x} dx \leq 1 = U_f(P)$. We can drop the equalities
because the lower & upper sums miss some area and include
some extra area (just look at the picture). Alternatively, do what
the book suggests and use $P = \{1, \frac{3}{2}, 2\}$.

9.12 Given F , find (a) $F'(-1)$ (b) $F'(0)$ (c) $F'(1/2)$ (d) $F''(x)$

(9) $F(x) = \int_0^x \frac{1}{t^2+9} dt$

$$F'(x) = \frac{1}{x^2+9}$$

$$F''(x) = \frac{-2x}{(x^2+9)^2}$$

(a) $F'(-1) = \frac{1}{(-1)^2+9} = \frac{1}{10}$ (b) $F'(0) = \frac{1}{0^2+9} = \frac{1}{9}$ (c) $F'(1/2) = \frac{1}{(1/2)^2+9} = \frac{4}{37}$

(d) $F''(x) = \frac{-2x}{(x^2+9)^2}$

(12) $F(x) = \int_1^x \sin(\pi t) dt$

$$F'(x) = \sin(\pi x)$$

$$F''(x) = \pi \cos(\pi x)$$

(17) Find $F'(x) = \frac{dF}{dx}(x)$ if $F(x) = \int_0^{x^3} t \cos(t) dt$

If $G(u) = \int_0^u t \cos(t) dt$, then $\frac{dG}{du}(u) = G'(u) = u \cos(u)$

Since $F(x) = G(x^3)$, then $\frac{dF}{dx}(x) = \frac{d}{dx}(G(x^3))$

$$\begin{aligned} \text{(use the chain rule)} \rightarrow &= \left(\frac{dG}{du}(x^3) \right) \left(\frac{d}{dx}(x^3) \right) \\ &= (x^3 \cos(x^3)) (3x^2) \\ &= 3x^5 \cos(x^3) \end{aligned}$$

5.3

$$\textcircled{1} \int_0^1 (2x-3) dx$$

Since $\frac{d}{dx}(x^2-3x) = 2x-3$, then

$$\int_0^1 (2x-3) dx = \left[x^2-3x \right]_0^1 = (1-3) - (0-0) = -2$$

$$\textcircled{2} \int_1^2 (2x+x^2) dx$$

Since $\frac{d}{dx}\left(x^2 + \frac{x^3}{3} + 17\right) = 2x + x^2$, then

$$\int_1^2 (2x+x^2) dx = \left[x^2 + \frac{x^3}{3} + 17 \right]_1^2 = \left(4 + \frac{8}{3} + 17 \right) - \left(1 + \frac{1}{3} + 17 \right) = 5\frac{1}{3}$$

$$\textcircled{11} \int_1^2 \left(3t + \frac{4}{t^2} \right) dt$$

Since $\frac{d}{dt}\left(\frac{3}{2}t^2 - \frac{4}{t}\right) = 3t + \frac{4}{t^2}$, then

$$\int_1^2 \left(3t + \frac{4}{t^2} \right) dt = \left[\frac{3}{2}t^2 - \frac{4}{t} \right]_1^2 = (6-2) - \left(\frac{3}{2} - 4 \right) = 6\frac{1}{2}$$

$$\textcircled{25} \int_0^{\pi/2} (\cos x) dx$$

Since $\frac{d}{dx}(\sin x) = \cos x$, then $\int_0^{\pi/2} (\cos x) dx = \sin x \Big|_0^{\pi/2} = 1 - 0 = 1$

$$\textcircled{31} \int_0^{2\pi} (\sin x) dx$$

Since $\frac{d}{dx}(-\cos x) = \sin x$, then $\int_0^{2\pi} (\sin x) dx = -\cos x \Big|_0^{2\pi} = (-1) - (-1) = 0$.