

Conceptual issues:

Graphical interpretation of a limit. English description of limit.

Definition of continuity. Visual understanding of continuity

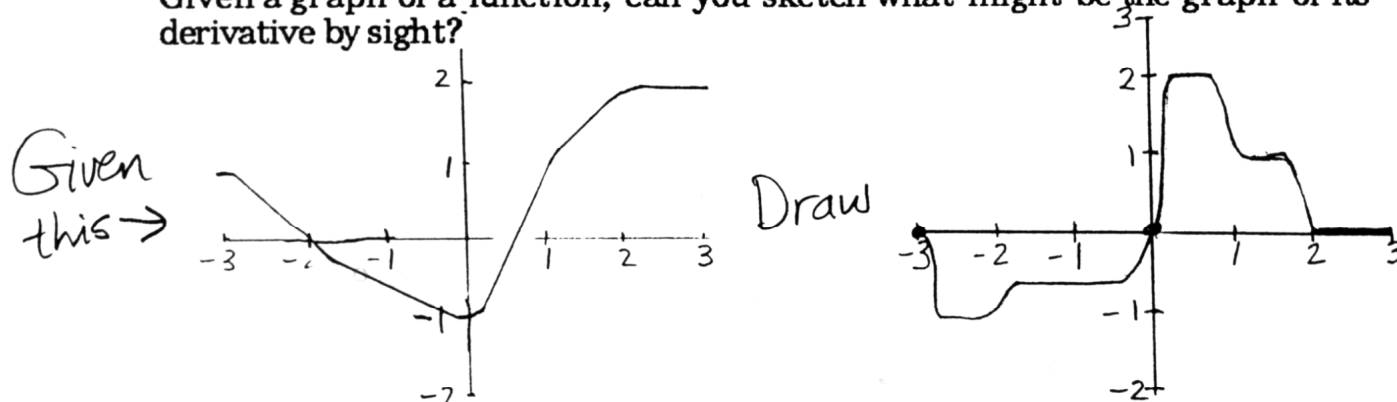
Definition of derivative.

Being able to take the derivative from the definition, for example, if $f(x) = x^2$, using the definition compute $f'(3)$.

What does a differentiable function look like magnified at a point of the graph? Why?

How is the derivative related to the graph of a function? Answer: It is the slope of the tangent line. Why does the definition of derivative tell you this?

Given a graph of a function, can you sketch what might be the graph of its derivative by sight?



If a function is not continuous at a point, why will the derivative not exist there?

Be able to interpret the derivative in English. In other words, if $f(t)$ gives the position of a moving car on a straight road, then $f'(t)$ gives the instantaneous velocity at time t . Why does the definition of derivative tell us this? Another question: Suppose $p(t)$ measures the distance from the capitol building as you drive around Austin. So you are not driving in a straight line. Now what does $p'(t)$ represent? Would it be the speed of the car or not? Another interpretation question: Suppose $A(r)$ is the area of a sphere of radius r , what does $A'(r)$ represent? etc.

Computational skills:

What is $\lim f(x)$ for some functions like in Exercises 2.1-2.3, but no ϵ - δ stuff.

Find the derivative of functions using product, quotient, chain rule. Functions could involve trig functions as well as polynomials.

Find dy/dx in terms of x and y by implicit differentiation.

Re-test

Calculus I (M408C) Name _____

~~Sept. 27~~ ^{Oct. 12}, 2000

Show work for each question. Put answers in boxes. In circle, put the number of pts you received on that question on the first test

1. Evaluate the following limits. If a limit does not exist, explain.

(4pts) i. $\lim_{x \rightarrow 0} \frac{x^2 + 1}{x + 2}$ ~~$\lim_{x \rightarrow 2} \frac{3x - 2}{x + 2}$~~

1st
test

(4pts) ii. $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$ ~~$\lim_{x \rightarrow 0} (x - \frac{4}{x})$~~

(4pts) iii. $\lim_{x \rightarrow 2} \frac{|x - 2|}{x - 2}$ ~~$\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$~~

(4pts) iv. $\lim_{x \rightarrow 0} \frac{\sin^2 3x}{5x^2}$ ~~$\lim_{x \rightarrow 0} \frac{\sin^2 5x}{3x}$~~ =

(4pts) v. $\lim_{x \rightarrow 2} f(x)$

where $f(x) = \begin{cases} x^3 & \text{if } x \neq 2 \\ 4 & \text{if } x = 2 \end{cases}$

(4pts) vi. $\lim_{h \rightarrow 0} \frac{\cos(3+h) - \cos 3}{h}$

(hint: Do you recognize this limit?)

$\rightarrow \lim_{h \rightarrow 0} \frac{\sqrt{3+h} - \sqrt{3}}{h} =$

2. Differentiate the following functions:

Do not simplify your answer.

(6pts) i. ~~$y = \sin^4(x^3 + 6x + 2)$~~

$$y = \cos^3((x^3 + 6x)^4)$$

1st
test
score

(6pts) ii. ~~$f(x) = \frac{x^2 + 2\sqrt{x}}{5}$~~

$$f(x) = \frac{x^4 + 2}{x^6 + 3x^4 + 6}$$

(6pts) iii. ~~$f(x) = (x^4 + 11x^2 + 1)(8x^2 + 5x)^5$~~

$$f(x) = (\sqrt{x})(\sin(x^2 + 2x + 1))$$

(6pts) 4. Use implicit differentiation to find $\frac{dy}{dx}$ if

~~$x^4 + 9x^3y^4 = 0$~~ $3x^2y^3 - 4y^2\sin x = 0$

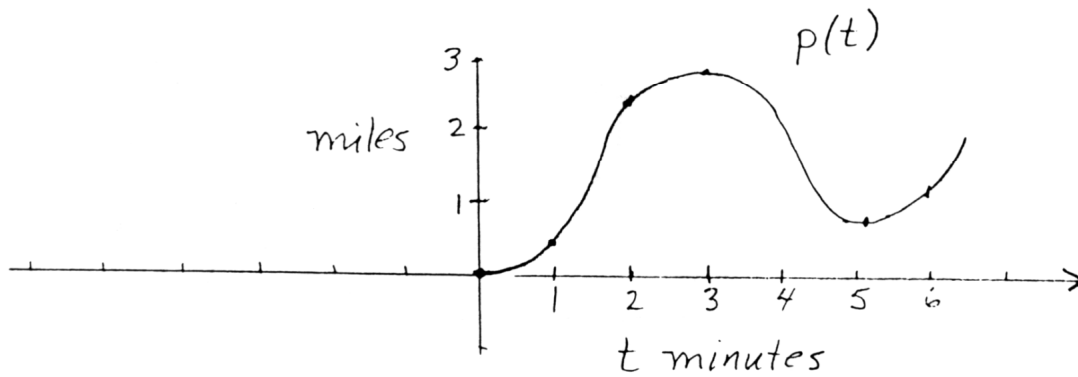
$\frac{dy}{dx} =$

5. Consider $f(x) = \frac{1}{x}$ ~~$f(x) = \frac{1}{x}$~~

(5pts) a. State the definition of $f'(3)$.

(10pts) b. Use the definition to compute $f'(3)$.

6.



The graph above tells the position of a car moving on a straight road.

(6pts) (a) Roughly how fast is the car going (the instantaneous velocity) at time $t=2$, $t=5$, and $t=6$.

$t=2$

$t=5$

$t=6$

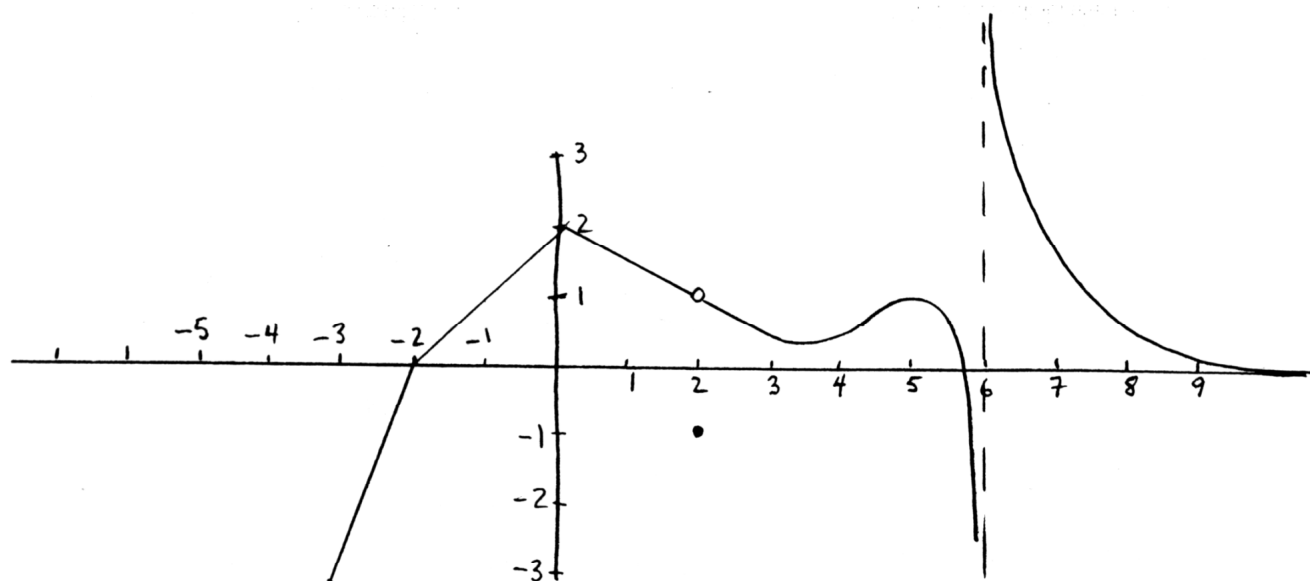


(5pts) (b) For time $t=2$, explain your reasoning in part (a) and relate it to the derivative,



(5pts) (c) At time $t=5$ is the car speeding up or slowing down? Same question for time $t=6$. Explain.





(6pts) (a) For what values of x is $f(x)$ not continuous? Classify each discontinuity.

(6pts) (b) For what values of x is $f(x)$ not differentiable? Explain briefly why for each place.

(9pts) (c) On the axes above, graph $f'(x)$ wherever it exists.

LIMITS

$$1) \lim_{x \rightarrow -3} \frac{x+3}{\sqrt{(x+3)^2}} =$$

$$2) \lim_{x \rightarrow 5} \frac{|x|}{|x|-5} =$$

$$3) \lim_{x \rightarrow 3} f(x) =$$

$$f(x) = \begin{cases} x-3, & x > 3 \\ 3, & x = 3 \\ x^2 - 4x - 5, & x < 3 \end{cases}$$

$$4) \lim_{x \rightarrow 0} \frac{1}{x^2} =$$

$$5) \lim_{x \rightarrow 2} f(x)$$

$$f(x) = \begin{cases} x^2 + 2, & x < 2 \\ x - 2, & x > 2 \end{cases}$$

$$6) \lim_{h \rightarrow 0} \frac{(\cos 4 + x + h) - (\cos 4 + x)}{h}$$

$$7) \lim_{y \rightarrow 0} \frac{\tan(x+y) - \tan x}{y}$$

$$8) \lim_{x \rightarrow 3} \frac{x^4 - 81}{x - 3}$$

$$9) \lim_{x \rightarrow 1/2} \frac{2x^5 - x^4}{4x^2 - 1}$$

$$10) \lim_{x \rightarrow 7} \frac{1}{x - 7}$$

DERIVATIVES

Implicit:

$$1) \cos(x+y) - 2x^2y = 0$$

$$2) x^3y^4 - \frac{1}{y} = (x^2+3)(y-4)$$

$$3) 4yx + 2000 - \frac{1}{x^2} = \frac{y+5}{4}$$

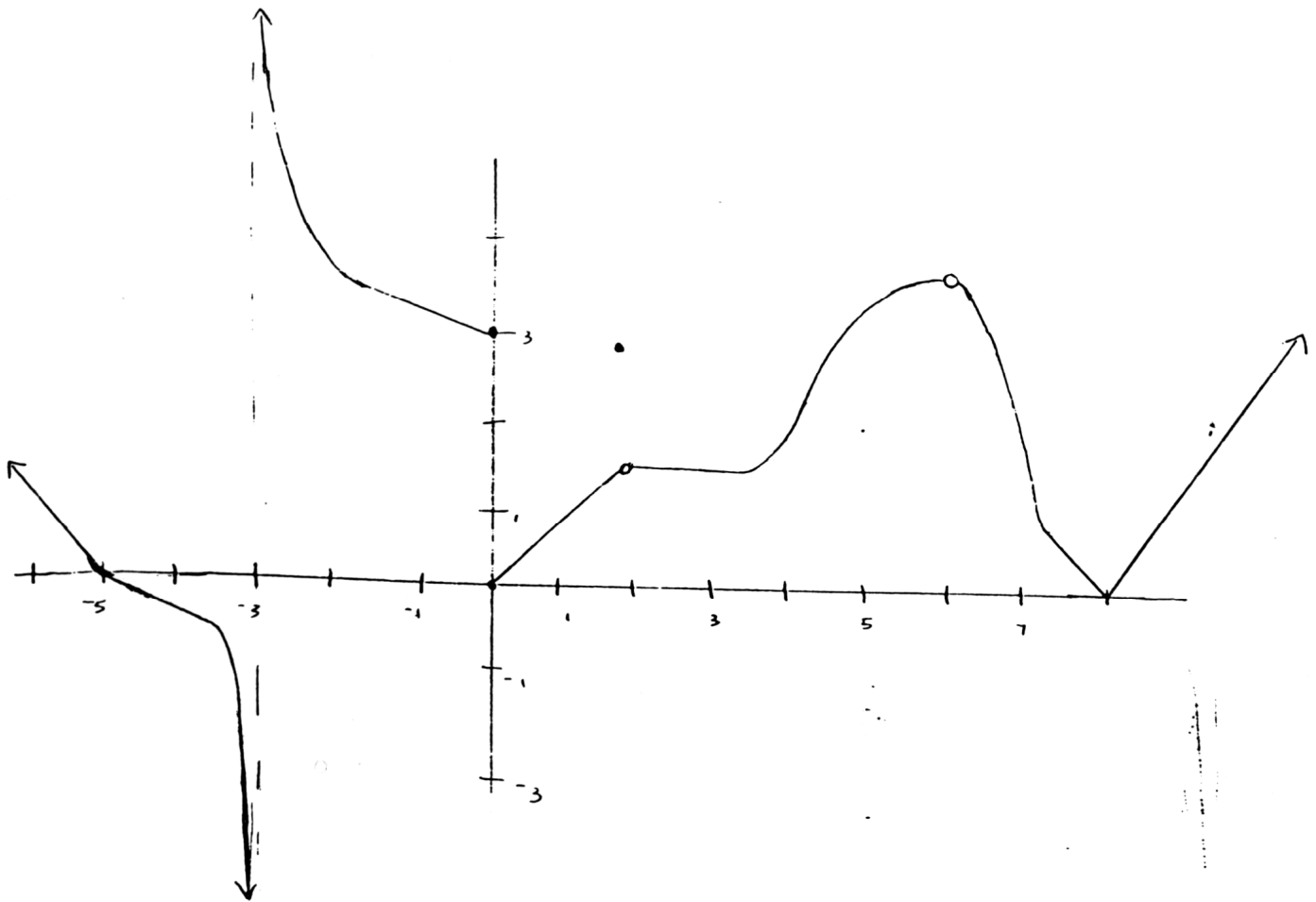
Using the
Definition:

$$1) \text{ Find } f'(x) \text{ when } f(x) = \frac{1}{x}$$

$$2) \text{ Find } f'(x) \text{ when } f(x) = \sqrt{x}$$

$$3) \text{ Find } f'(5) \text{ when } f(x) = x^2 + 7$$

Understanding Discontinuities & Differentiation.

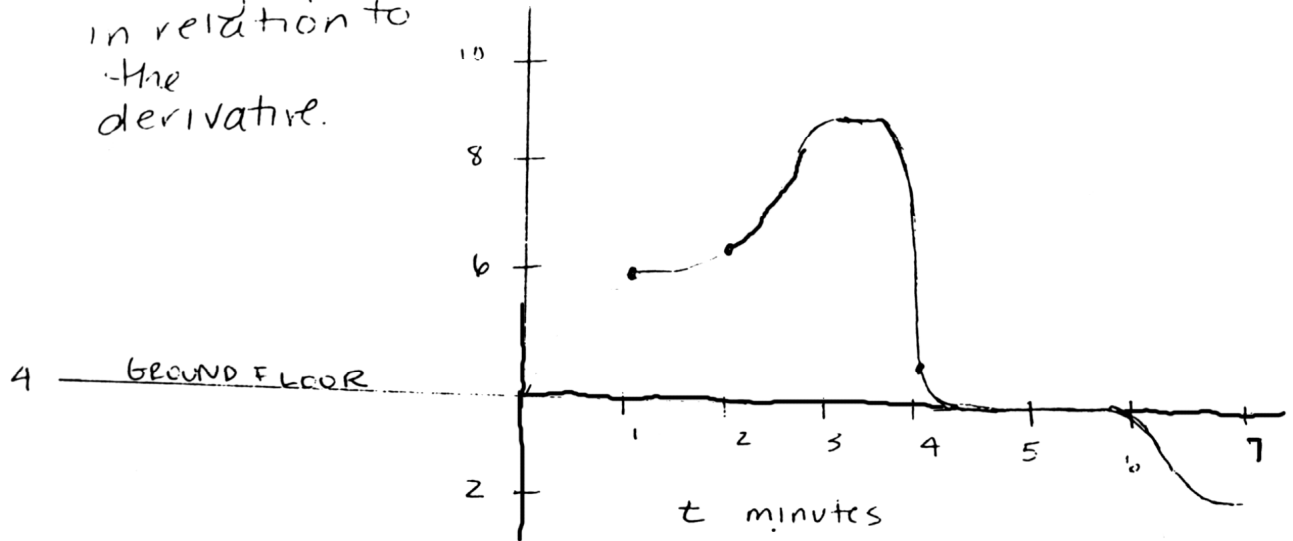


(1) Where is $f(x)$ discontinuous? Classify the discontinuities.

(2) For what values of x is $f(x)$ not differentiable? Explain your reasoning.

(3) Sketch the graph of the derivative on the axis above.

Interpreting
the graph
in relation to
the
derivative.



The graph above indicates the movement of the elevator
in RLM. (in off-peak hours)

(a) tell how quickly the elevator is going at time
 $t = 1$? $t = 2$ $t = 4$

b) describe your reasoning at each of these
times, and relate it to the derivative.

c) give an overall explanation of what the elevator
is doing and indicate where it is speeding up
and slowing down (explain your reasoning).

Other Derivatives :

Find $f'(x)$

$$1) f(x) = 5 \csc^3(x^3 - 3 - x)$$

$$2) f(x) = (\sqrt{x - 4x^5})^6$$

$$3) f(x) = \sqrt[3]{(3x + 5x^2 - \frac{1}{x})^5}$$

$$4) (7x - 5x^4 + 2)(4x^3 + 6x^2 - 4x) = f(x)$$

$$5) f(x) = \frac{1}{(5x^2 - 4)}$$

$$6) f(x) = \frac{\sqrt{x} - 2x}{7}$$

$$7) f(x) = \left(\frac{1}{\sqrt{x}} \right) \left(\frac{5x^2}{4} \right)$$