$$\begin{bmatrix} A & B & 0 \\ B & C & D \\ 0 & D & E \end{bmatrix}$$

$$(A-\lambda)(C-\lambda)(E-\lambda) - D^2(A-\lambda) - B^2(E-\lambda) = 0$$

$$-\lambda^3 + (A+C+E)\lambda^2 + (B^2+D^2-AC-AE-CE)\lambda$$
$$+ (ACE - AD^2 - EB^2) = 0$$

$$-(\lambda-\alpha)^2(\lambda-\beta)^2$$

$$-\lambda^3 + (2\alpha+\beta)\lambda^2 - (\alpha^2+2\alpha\beta)\lambda + \alpha^2\beta$$

$$2\alpha+\beta = A+C+E$$

$$-\alpha(\alpha+2\beta) = B^2+D^2-AC-AE-CE$$

$$\alpha^2\beta = ACE - EB^2 - AD^2$$

$$\beta = (A+C+E) - 2\alpha$$

$$-\alpha\left(\alpha + 2(A+C+E) - 4\alpha\right) = \underbrace{B^2 + D^2 - AC - AE - CE}_{RHS}$$

$$3\alpha^2 - 2(A+C+E)\alpha - RHS = 0$$

$$\alpha = \frac{2(A+C+E) \pm \sqrt{4(A+C+E)^2 + 12\,RHS}}{6}$$

$$= \frac{A+C+E}{3} \pm \sqrt{\left(\frac{A+C+E}{3}\right)^2 + \frac{RHS}{3}}$$

Let $3p = A + C + E$

$m^2 = p^2 + n \quad w/\ 3n = RHS$

So $RHS = 3(m^2 - p^2)$

and $\alpha = p \pm m$

$\beta = 3p - 2(p \pm m) = p \mp 2m$

$$RHS = B^2 + D^2 - AC - AE - CE$$

$$= B^2 + D^2 - AC - E(A+C)$$

$$- A(C+E) - CE$$

$$- A(3p - A) - C(3p - (A+C))$$

$-A(3p-A) - C(3p-A) + C^2$

$C^2 - (3p-A)(A+C)$

$-3p(A+C)$

$+A^2$

$+C(A+C)$

$(C-3p)(A+C) + A^2$

p, m, A, C free

$$\alpha = p \pm m$$

$$\beta = p \mp 2m$$

$$E = 3p - (A+C)$$

$$+ AC + AE + CE$$

$$B^2 + D^2 = 3(m^2 - p^2) + (3p - C)(A+C) - A^2$$

$$(3p - A)(A+C) - C^2$$

$$EB^2 + AD^2 = ACE - \alpha^2\beta$$