Appendix III

The Leap Frog Formulas

Choose a positive time step Δt and let $t_k = k\Delta t, k = 0, 1, 2, \ldots$. For $i = 1, 2, 3, \ldots, N$, let P_i have mass m_i and at t_k let it be at

$$\vec{r}_{i,k} = (x_{i,k}, y_{i,k}, z_{i,k});$$

have velocity

$$\vec{v}_{i,k} = (v_{i,k,x}, v_{i,k,y}, v_{i,k,z}),$$

and have acceleration

$$\vec{a}_{i,k} = (a_{i,k,x}, a_{i,k,y}, a_{i,k,z}).$$

The leap frog formulas, which relate position, velocity and acceleration are

$$\frac{\vec{v}_{i,1/2} - \vec{v}_{i,0}}{(1/2)\Delta t} = \vec{a}_{i,0}, \qquad \text{(Starter)} \tag{1.10}$$

$$\frac{\vec{v}_{i,k+1/2} - \vec{v}_{i,k-1/2}}{\Delta t} = \vec{a}_{i,k}, \qquad k = 1, 2, 3, \ldots, \tag{1.11}$$

$$\frac{\vec{r}_{i,k+1} - \vec{r}_{i,k}}{\Delta t} = \vec{v}_{i,k+1/2}, \quad k = 0, 1, 2, 3, \ldots, \tag{1.12}$$

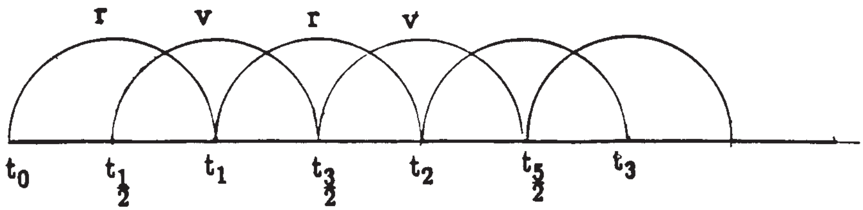

Figure A3.1. Leap frog.

or, explicitly,

$$\vec{v}_{i,\frac{1}{2}} = \vec{v}_{i,0} + \frac{1}{2}(\Delta t)\vec{a}_{i,0}, \qquad \text{(Starter)} \tag{1.13}$$

$$\vec{v}_{i,k+\frac{1}{2}} = \vec{v}_{i,k-\frac{1}{2}} + (\Delta t)\vec{a}_{i,k}, \quad k = 1, 2, 3, \ldots, \tag{1.14}$$

$$\vec{r}_{i,k+1} = \vec{r}_{i,k} + (\Delta t)\vec{v}_{i,k+\frac{1}{2}}, \quad k = 0, 1, 2, 3 \ldots. \tag{1.15}$$

Note that (1.11) and (1.12) are two point central difference formulas. The name *leap frog* derives from the way position and velocity are defined at alternate, sequential time values . As shown in the Figure A3.1, the r values are defined at the times $t_0, t_1, t_2, t_3, \ldots$, while the v values are defined at the times $t_{\frac{1}{2}}, t_{\frac{3}{2}}, t_{\frac{5}{2}}, t_{\frac{7}{2}}, \ldots$. The figure also symbolizes the children's game "leap frog".